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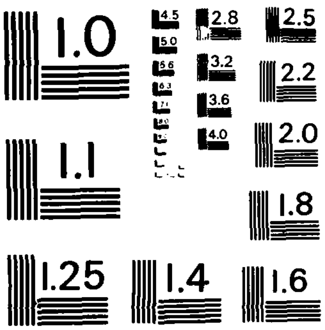

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NRL Report 8721

NRL Reverberation Model: A Computer Program for the Prediction and Analysis of Medium- to Long-Range Boundary Reverberation

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The Naval Research Laboratory has developed a comprehensive simulation model to predict boundary reverberation from either the ocean surface or bottom. To date the model has been utilized principally for active surveillance applications in which long ranges, typically 100 to 500 nmi, and low frequencies, 50 to 500 Hz, have been emphasized. This model is not restricted, however, to long range or low frequency applications. The model predicts boundary reverberation for both monostatic and bistatic active system geometries. Among the unique

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20. ABSTRACT (Continued)

capabilities of the NRL reverberation model are: the treatment of both monostatic and bistatic geometries; the inclusion of all multipath contributions in the reverberation calculation; range-dependent propagation effects (presently for monostatic geometries); angle-dependent surface and bottom scattering strength functions; frequency spreading of reverberation returns induced by forward scattering at the sea surface; and the doppler-shifting of surface backscattered energy. The frequency spreading and doppler-shifting estimates are calculated by supplemental programs which are not presently included in the core reverberation program to be described in this report.

The reverberation model comprises a sequence of computer programs. This allows the evaluation of intermediate outputs, and facilitates the model's application to parameter studies by delaying parameter inputs until the calculation stage in which they are needed is reached. In the first stage, sound speed profiles and bottom elevations are input as a function of range. Normal dimensioning permits for up to 50 sound speed profiles and 50 bottom elevations but is easily expanded to cover more refined environmental specification. Next, a ray-acoustic propagation model is used to trace out rays from both source and receiver, keeping track of various parameters at each boundary encounter for each ray path. Signal intensity is modified by bottom losses, geometrical spreading, and volume absorption which varies as a function of frequency. The number of acoustic rays traced is a variable and is selected to ensure adequate sampling of the propagation field. Then, the reverberation contribution from each elemental scattering area on the boundary is obtained by calculating the time-dependent contributions due to all possible reverberant paths from source to scattering area to receiver. That is, the calculation utilizes all m outgoing paths combined with n backscattered return paths. Each contribution is weighted by source and receiver beam patterns, the angle-dependent surface or bottom backscattering strength values, and the duration and shape of the source pulse. Finally, by integrating over the entire scattering surface, the total boundary reverberation is determined. The main outputs of the reverberation model are average reverberation *vs* time plots and average reverberation density as a function of vertical angle at selected times. When frequency spreading and doppler-shifting calculations are included, it is also possible to plot reverberation spectral-time histories.

Presented are

~~This report presents~~ the theoretical foundations of the reverberation model, the numerical implementation of this theory, and a detailed description of actual program execution. A complete description of all inputs and outputs is included. Finally, examples illustrating the use of the sequence of programs are provided.

CONTENTS

NRL REVERBERATION MODEL: A COMPUTER PROGRAM FOR THE PREDICTION AND ANALYSIS OF MEDIUM- TO LONG-RANGE BOUNDARY REVERBERATION

INTRODUCTION

This document describes

A sequence of computer programs to predict long-range, low frequency monostatic or bistatic boundary reverberation from either the ocean surface or bottom has been developed. The formulation requires sets of acoustic rays to be traced out from both the source and receiver, keeping track of various parameters at each boundary encounter. The reverberation contribution from each elemental scattering area on the boundary is obtained by calculating the time-dependent contributions due to all possible reverberant paths from source to scattering area to receiver. That is, the calculation utilizes all outgoing paths combined with all backscattered incoming paths. Each contribution is weighted by the source and receiver beampatterns, the angle-dependent surface or bottom backscattering strength values, and the duration and shape of the source pulse. Finally, by integrating over the entire scattering surface, the total boundary reverberation is determined. Some of the more significant features of this computer model are: its ability to treat both monostatic and bistatic geometries; the capability of modeling the ocean environment as range dependent when the source and receiver are not separated horizontally; the ability to input source and receiver positions and beam patterns; the use of angle-dependent scattering coefficients at the surface and bottom; and the ability to select the duration and shape of the source pulse. It is also possible to include frequency spreading from forward surface scattering and doppler-shifted surface backscattering calculations. At present, the model is capable of handling a true bistatic geometry for a range-independent environment *only*. The main outputs of the model are plots of reverberation *vs* time, and reverberation as a function of vertical angle at selected times. Table 1 summarizes the input parameters for both monostatic and bistatic reverberation calculations.

The theoretical model is discussed in the first section. Computer implementation is then described. The development is similar to that given in a previous report [1]. The remainder of the report describes in detail the means of actually executing the computer program. A complete description of all inputs and outputs is included. Finally, examples illustrating the use of the sequence of programs are provided.

Table 1 — Reverberation Model Parameters

MONOSTATIC	BISTATIC
Source/Receiver Positions	Same
Source/Receiver Patterns	Same
Range Dependent Sound Speed	Constant Profile
Range Dependent Bottom Profile	Flat Bottom
Angle Dependent Bottom Loss Tables	Same
Angle Dependent Surface Scattering Strength	Same
Angle Dependent Bottom Scattering Strength	Same
Pulse Length, Selectable	Same

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THEORETICAL MODEL

To give a qualitative description of the way in which boundary reverberation arises, suppose that a point source S radiates a time-dependent signal into the three-dimensional ocean. The acoustic energy is propagated away from S along the ray paths. When a ray encounters an ocean boundary, it is assumed to continue to propagate in the direction of specular reflection, with a small amount of the incident radiation being reradiated or scattered according to some scattering law. The scattering is assumed to arise from the excitation of small scattering elements in the boundary, each of which acts as a weak secondary point source of radiation. Some of the scattered rays will arrive at the receiving point R, and the sum of these returns, as a function of time, is the boundary reverberation.

Because the frequency of the reverberation arising from surface scattering will usually be doppler shifted, and that from bottom scattering will not, we can calculate these two types of reverberation separately. Thus for either type of reverberation it will be necessary to trace a set of acoustic rays out from both S and R, keeping account of travel times, ray intensities, and other parameters at each boundary encounter. (If the receiving point R is coincident with the source point S, only one ray trace is necessary.) To develop a model of the reverberation process from which an expression for the mean reverberation decay can be deduced, three specific assumptions concerning the scattering process are made:

1. A small element of area dA on the boundary, when excited by a wave that is nearly plane, acts as a secondary point source for exactly the duration it is excited by the incident wave. Furthermore, the intensity of this secondary point source is proportional to the incident intensity and to dA, the constant of proportionality being the scattering strength σ.

2. For purposes of calculating a mean reverberation envelope, interference effects associated with differences in acoustic phase can be ignored. Thus each ray arrival at a scattering element produces a set of scattered rays, each having, in effect, a random phase shift relative to the incident ray. This leads to the determination of a mean reverberation envelope which is representative of ensemble-averaged reverberation returns. An envelope based on coherent summation of scattered paths would be more indicative of a single sample within an ensemble.

3. The scattering layer at the ocean surface can be approximated by a horizontal plane and appropriate scattering coefficient, and similarly at the bottom the scattering surface has approximately the same topography as the bottom, with its appropriate scattering coefficient.

The basic procedure for calculating reverberation when employing the concept of scattering strength is straightforward. For example, at an element of the ocean surface from which surface reverberation is to be calculated, there is associated a set of reverberant paths, each making an elemental contribution to the reverberation. Each elemental contribution is composed of the transmission losses associated with its reverberant path from the source to the scattering point and from the scattering point to the receiver, appropriately weighted by the source and receiver beam patterns, the scattering strength, and the area of the element. The expected or average value of the instantaneous reverberation is then the sum of all those elemental contributions active at that instant.

A general expression for calculating boundary reverberation can be formulated based on the assumptions outlined above. We will first derive the expression for monostatic boundary reverberation. The extension to bistatic reverberation will then be made.

Monostatic Boundary Reverberation

Figure 1 shows a source-receiver $S-R$ that ensonifies a scattering element dA via a propagation path i and receives reverberation from dA via, in this case, a different return path j. (The figure depicts surface reverberation. For bottom reverberation the coordinates of dA would be (X, Y, Z_b) where

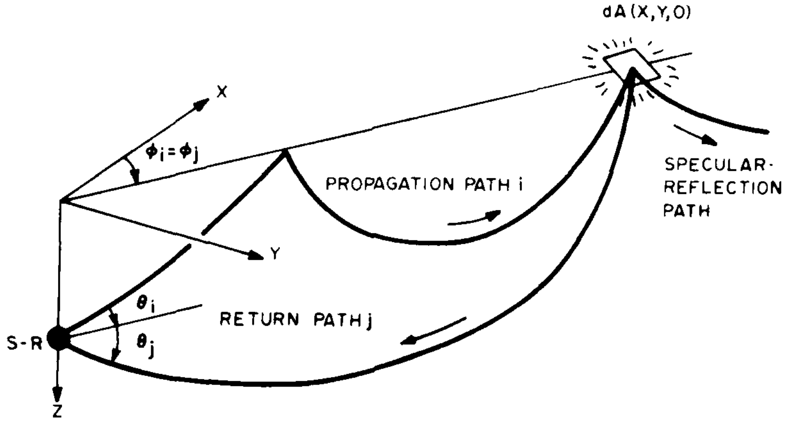

Fig. 1 — Source-receiver ensonifying a scattering element dA via propagation path *i* and receiving reverberation via return path *j*

Z_b is the ocean depth.) It is supposed that S emits a time-varying signal of intensity $I_1(t)$, weighted by a beam pattern B. This intensity is diminished by the transmission loss L_i along path i from S to dA. The incident intensity at dA is then weighted by the scattering strength σ. The reverberant return is further diminished by the transmission loss L_j along path j from dA to R. At R it is weighted by the receiver beam pattern $\bar{B}$. Further, the reverberation return is delayed in time by the two-way travel time $T_i + T_j$. That is, the unweighted initial source intensity which gives rise to the reverberation contribution at time t due only to outward path i and return path j is $I_1(t - T_i - T_j)$. By summing over all the multipath combinations between $S-R$ and dA, the total contribution from dA is obtained. These intensities will be summed with regard to time but without regard to phase, owing to the second assumption. Finally by integrating over the area A of scattering points, the total reverberation $R(t)$ at time t is given by

$$R(t) = \int\int_A \sum_{i \in m(X,Y)} \sum_{j \in m(X,Y)} I_1(t - T_i(X, Y) - T_j(X, Y)) \frac{B(\theta_i, \phi_i)\bar{B}(\theta_j, \phi_j)}{L_i(X,Y)L_j(X,Y)} \sigma(\theta_i', \phi_i, \theta_j', \phi_j)\, dA, \tag{1}$$

where $m(X,Y)$ indexes the set of rays between $S-R$ and dA. Note that the scattering strength $\sigma(\theta_i', \phi_i, \theta_j', \phi_j)$ in Eq. (1) depends on the inclination and azimuth angles of incidence and scatter; that is, at this level of generality, a full three-dimensional scattering pattern would be employed.

It is not possible to numerically evaluate Eq. (1) directly due to the lack of a three-dimensional propagation model. Therefore the monostatic reverberation model was reduced to two dimensions by assuming

1. cylindrical symmetry of the ocean environment about the Z-axis (with the cylindrical coordinates being (x,z,ϕ))

2. directions of reflected and scattered energy are confined to the vertical $x - z$ plane containing the incident ray path.

It is also assumed that the beam patterns of source and receiver are both separable, and we write

$$B(\theta, \phi) = a(\phi)b(\theta), \tag{2}$$

$$\bar{B}(\theta, \phi) = \bar{a}(\phi)\bar{b}(\theta). \tag{3}$$

Let

$$\Phi = \int_0^{2\pi} a(\phi)\bar{a}(\phi)\, d\phi \tag{4}$$

be the equivalent azimuthal beamwidth of $S-R$. Then Eq. (1) becomes

$$R(t) = \Phi \int_0^\infty \sum_{i\in m(x)} \sum_{j\in m(x)} I_1(t - T_i(x) - T_j(x)) \frac{b(\theta_i)\bar{b}(\theta_j)}{L_i(x)\, L_j(x)} \sigma(\theta_i', \theta_j')x\, dx. \tag{5}$$

Next, we take the signal emitted by S to be a wave of constant intensity I_1 and duration D that is turned on at S at $t = 0$. Then only those reverberating elements at ranges x satisfying

$$0 \leqslant t - T_i(x) - T_j(x) \leqslant D \tag{6}$$

are active in producing reverberation at R at time t. The double inequality of Eq. (6) is equivalent to

$$t - D \leqslant T_i(x) + T_j(x) \leqslant t. \tag{7}$$

Therefore the set $X_{ij}(t)$ of reverberators active via the path pair (i, j) at time t is defined by

$$X_{ij}(t) = \left\{ x \middle| t - D \leqslant T_i(x) + T_j(x) \leqslant t \right\}. \tag{8}$$

The characteristic function $C\chi$ of a set χ may be defined by

$$C_\chi(x) = \begin{Bmatrix} 1 & x \in \chi \\ 0 & x \notin \chi \end{Bmatrix}. \tag{9}$$

The reverberation $R(t)$ at R can then be written as

$$R(t) = I_1\Phi \int_0^\infty \sum_{i\in m(x)} \sum_{j\in m(x)} C_{X_{ij}(t)}(x) \frac{b(\theta_i)\, \bar{b}(\theta_j)}{L_i(x)\, L_j(x)} \sigma(\theta_i', \theta_j')\, x\, dx. \tag{10}$$

It has been shown [2] that this formulation reduces to the standard result [3] when the sound speed c and scattering coefficient σ are constant and the source-receiver is placed at a depth z_0 below the surface in a bottomless ocean. That result, for times $t >> D + 2(z_0/c)$, is

$$R(t) \approx \sigma I_1 \Phi \left(\frac{cD}{2}\right) \left(\frac{ct}{2}\right)^{-3}. \tag{11}$$

In terms of the path length $l = ct/2$,

$$R(l) \approx \sigma \frac{I_1}{l^4} \left[\left(\frac{cD}{2}\right) l\Phi\right], \tag{12}$$

the quantity in the brackets being the reverberating area. It is straightforward to extend this result to the case of multiple reflections in a channel with a bottom at a finite depth.

Equation (10) is a generalization of the isospeed result within the limitations of the assumptions made in deriving it. The generalization is twofold. First, Eq. (10) can accommodate a wide class of sound-speed fields, including horizontal as well as vertical gradients, provided the assumption of cylindrical symmetry is reasonably well met over the significant azimuthal directions defined by the source-receiver beam patterns. Thus refractive effects are taken into account. Second, reverberation arising from different outgoing and incoming paths is included in addition to reverberation arising only

from coincident outgoing and incoming paths. At shorter ranges the "noncoincident" contributions are probably not appreciable, but at longer ranges they may be significant even with vertically directive beam patterns.

Bistatic Boundary Reverberation

Equation (1) may be generalized to the case where the source and receiver are separated in range (the bistatic case). The times at which an elemental contribution of the boundary to bistatic reverberation is active are determined by the travel times along the propagation and return paths and the duration of the transmitted pulse. Again we take the emitted signal to be a wave of constant intensity I_1 and duration D that is turned on at S at $t = 0$. Then, a scattering point is actively contributing at time t via a propagation path i and a return path j if

$$t - D \leqslant T_i(X,Y) + T_j(X,Y) \leqslant t, \tag{13}$$

where T_i and T_j are the one way travel times along the paths i and j. The set of points active at time t via the path pair (i,j) is then:

$$A_{ij}(t) = \{(X,Y)\,|\,t - D \leqslant T_i(X,Y) + T_j(X,Y) \leqslant t\}. \tag{14}$$

The reverberation $R(t)$ from the boundary area A can then be written:

$$R(t) = I_1 \iint_A \sum_{i \in m(X,Y)} \sum_{j \in n(X,Y)} C_{A_{ij}(t)}(X,Y) \frac{B(\theta_i, \phi_i)\, \overline{B}(\theta_j, \phi_j)}{L_i(X,Y)\, L_j(X,Y)} \sigma(\theta_i', \phi_i, \theta_j', \phi_j)\; dA, \tag{15}$$

where the characteristic function $C_{A_{ij}(t)}(X,Y)$ is 1 if $(X,Y) \in A_{ij}(t)$ and 0 otherwise, B is the source beam pattern applied to path i, L_i is the accumulated loss along path i to the scattering point, L_j is the accumulated loss along path j from the scattering point to the receiver, $\overline{B}$ is the receiver beam pattern applied to path j, and σ is a fully three-dimensional scattering strength. Note that, in Eq. (15), $m(X,Y)$ indexes the set of rays between the source S and dA, while $n(X,Y)$ indexes the set of rays between the receiver R and dA. For a bistatic reverberation calculation, these two index sets are not necessarily the same. This reflects the fact that a given scattering point generally will be at different ranges from the source and receiver owing to the horizontal source-receiver separation, and therefore a different set of rays will connect the scattering point to the source than to the receiver.

COMPUTER IMPLEMENTATION

A four stage sequence of computer programs was developed to numerically calculate boundary reverberation. The first three stages provide preliminary calculations necessary for the evaluation of Eqs. (10) or (15). These equations are numerically evaluated in the fourth stage. The process is illustrated by the block diagram in Fig. 2.

Implementation of the four stage sequence of computer programs for calculating bistatic reverberation is as follows:

1. The medium is described by providing a sound speed profile and a bottom depth to the computer program representing the first stage. The sound speed profile consists of discrete depths for which the sound speed is specified. An output file is produced.

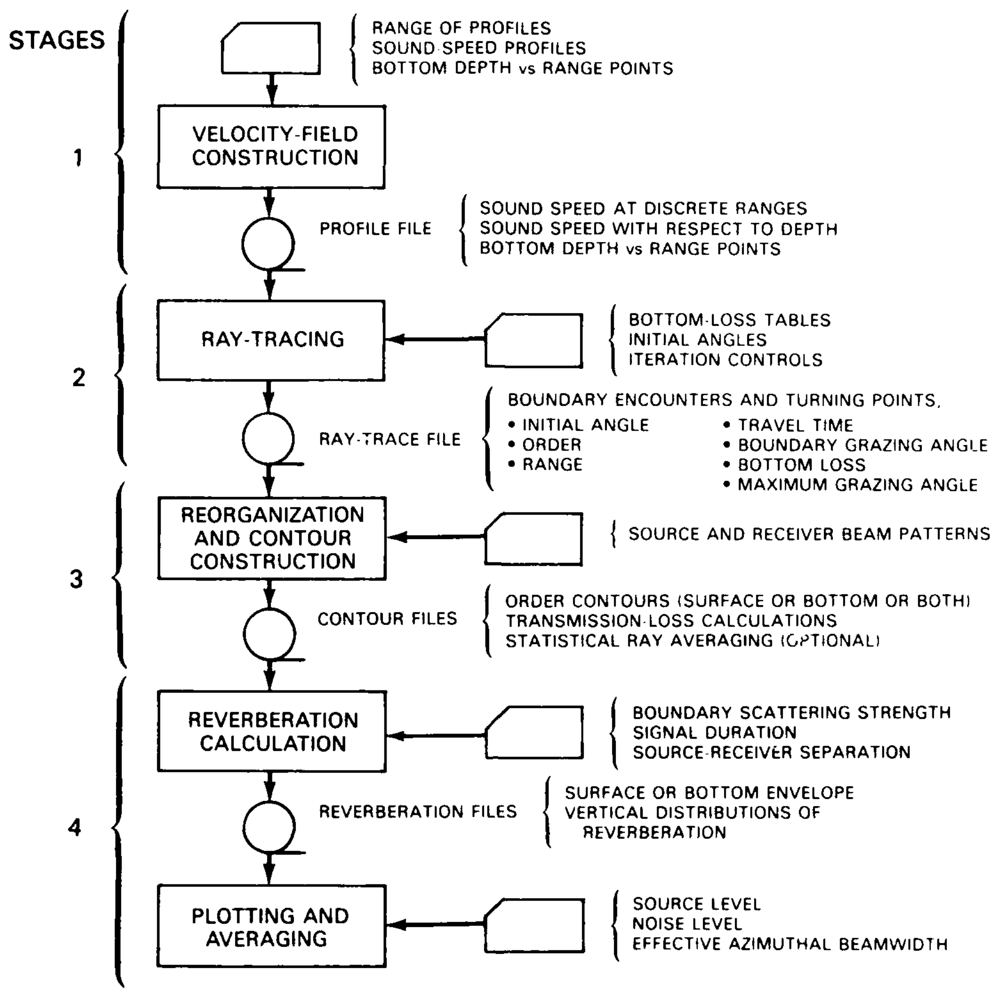

Fig. 2 — Four-stage sequence of computer programs to calculate boundary reverberation

2. A ray tracing program is executed to calculate all boundary hits associated with rays that are traced. The input to this program consists of the file just created plus additional card inputs which specify ray-tracing parameters such as initial angles of rays to be traced, various iteration controls, and bottom-loss data. Bottom-loss information is specified in the form of tables of grazing angle *vs* dB loss.

The ray-tracing program is run twice, tracing rays from the source to the boundaries and then tracing rays from the receiver to the boundaries. Each execution produces an output data file containing information on the location and character of the ray reversals encountered during ray tracing. By character is meant ray order (see below), initial angle of the ray, travel time, ray grazing angle at a boundary encounter, and accumulated bottom loss along the ray.

3. The ray-tracing results are reorganized, and the transmission loss along each ray is calculated at each computed boundary hit. Surface and bottom-hit data are processed separately. Volume absorption and geometric-spreading loss are combined with the accumulated bottom loss to determine the transmission loss for each ray. At this point the caustic correction is applied if desired. These transmission-loss values are weighted by the source or receiver vertical beam pattern. The results of the reorganization and the computed transmission losses are written onto data files.

4. The data files containing the reorganized ray-tracing results are input to a program that calculates the received bistatic boundary reverberation from either the surface or bottom at a discrete set of times by means of Eq. (15). The boundary scattering strength is specified in tabular form as a function of incident and backscattered angles. Although not available in the version of the reverberation production model reported here, the authors have expanded the reverberation calculation to include estimates of both doppler shifting of surface reverberation and frequency spreading of all surface forward scattered returns in either surface or bottom reverberation. This extension of the model will be reported elsewhere.

The basic reverberation program produces the calculated reverberatio[illegible]velope *vs* time on another data file. In turn this file is used by a program that plots the predicted [illegible]ve after adjusting for source level and noise level. The plotting program is also capable of averagi[illegible] several curves and of time averaging. It is also possible to obtain the vertical angular arrival structure [illegible] the reverberation at a set of discrete times. Future inclusion of doppler and spreading effects will pr[illegible]e the additional capability of producing a series of intensity *vs* frequency *vs* time plots.

The bistatic calculation can be collapsed to compute "monostatic" or "pseudo monostatic" reverberation characteristics by appropriate selection of input parameters. Monostatic refers to the usual case of coincident source and receiver. The case where source and receiver are separated in depth but not horizontally is referred to as pseudo monostatic. In either of these cases, a range-dependent environment may be specified by inputting to stage 1 a sequence of sound-speed profiles and bottom depths *vs* range. We now describe each of these four stages in more detail.

Sound-Speed Profile and Bottom Depth Contour

Before the actual ray tracing can be carried out, the sound-speed field and bottom topography must be modeled and written onto a file. This file serves as an input to program REVRAP, the ray-tracing program. The sound-speed field construction is accomplished by program PROFIL. If necessary the profile is extended linearly from the last depth input to the bottom depth. Program PLTENV creates Calcomp plots of the profiles and bottom topography as a function of range from the file generated by PROFIL.

Mathematical details of the sound-speed field and bottom topography construction are in [4, p. 52].

Ray Tracing

Program REVRAP reads the file created by program PROFIL and traces rays using a method based on a technique suggested by Hudson Laboratories [4]. The procedure is an iterative one in which a ray is incremented from point to point along its path. This is accomplished by evaluating Taylor-series expansions in arclength of various ray parameters such as range, depth, travel time, and ray angle. These expressions are derived from the basic ray equation:

$$\frac{d}{ds}\left[\frac{1}{c(x,z)}\,\frac{dP}{ds}\right] = \nabla\left[\frac{1}{c(x,z)}\right]. \qquad (16)$$

The sound speed $c(x, z)$ is assumed to be known at every range x and depth z throughout the two-dimensional medium; P is the position vector to a point on the ray, and s is arclength along the ray.

The ray-tracing model has the following features:

- Accommodates multiple profiles for possible horizontal variations in sound speed

- Each profile is defined by a table of sound speed *vs* depth, with weighted parabolic interpolation used between specified depths

- Assumes a linearly segmented ocean bottom and flat surface, with rays specularly reflecting from both boundaries

- Uses incremental finite-series approximations to ray paths

- Multiple bottom-loss tables may be applied to various range intervals

- Up to 1000 rays can be traced.

For the purpose of estimating surface and bottom reverberation, a set of rays including the source and receiver main beam are traced. These rays are chosen so that the resulting boundary hits approximate the true order contours accurately. If for any reason it is felt that an insufficient number of rays have been traced initially, additional ray traces may be performed and the results consolidated by using program MERGE.

The rays are traced through the medium which is described by multiple sound-speed profiles, a linearly segmented bottom, and a flat surface. They are traced one at a time between certain predetermined ranges, after which accumulated boundary-hit and turning-point (crest and valley) information is written onto a file. For each ray reversal, seven statistics are recorded:

- Initial angle of ray
- Order of the occurrence
- Range of the occurrence
- Travel time
- Ray grazing angle at boundary encounters or the depth of turning point
- Accumulated bottom loss
- Maximum grazing angle with which the ray has encountered the surface.

Specific details of the ray tracing iteration are in [4, pp. 52, 55-56]

Reorganization and Transmission-Loss Calculations

Each file created by an execution of program REVRAP serves as an input file to programs PLTCON and CONTUR. Program PLTCON provides a Calcomp plot of "order contours" as a function of initial angle and horizontal range by reordering the rays traced by program REVRAP. (See explanation of order contours below.) Program CONTUR also reorders the rays traced by REVRAP to form order contours. In addition program CONTUR computes the full transmission loss for each ray at every boundary encounter with an optional caustic correction and creates an output file. This output file is a necessary input file for program BISTRV which calculates reverberation.

Order Contours — We first describe the concept of order contours as they relate to a monostatic reverberation calculation. Consider the cyclic ray propagated from the point $S-R$ in Fig. 3. We adopt the convention of numbering the ray crests or surface hits with odd integers and using even integers for the valleys or bottom hits. The integers increase away from the source and are used to classify the ray

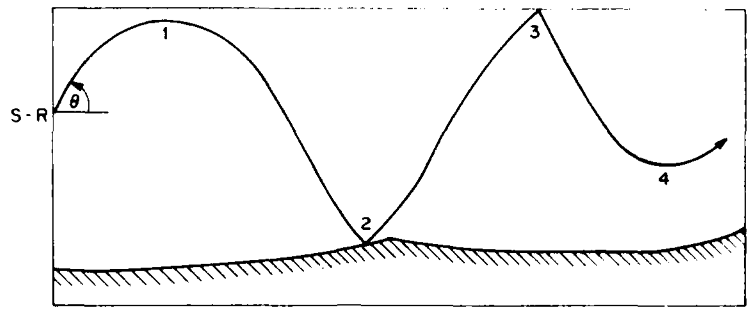

Fig. 3 — Numbering of ray crests, ray valleys, and boundary hits illustrated for one ray

by the number of oscillations it has made. Rays with the same number of oscillations are said to be of the same "order." Order contours then are derived from the ranges at which rays encounter a boundary. Ranges corresponding to ray reversals (i.e., ranges at which rays reverse their vertical direction) which are not due to boundary encounters do not define points on order contours. Order contours are illustrated in Fig. 4 where curves determined by the boundary encounters of a given order are plotted on an initial source-angle-*vs*-range coordinate system. A given contour need not necessarily be a continuous curve but may consist of several disjoint segments separated by intervals of ray turning points.

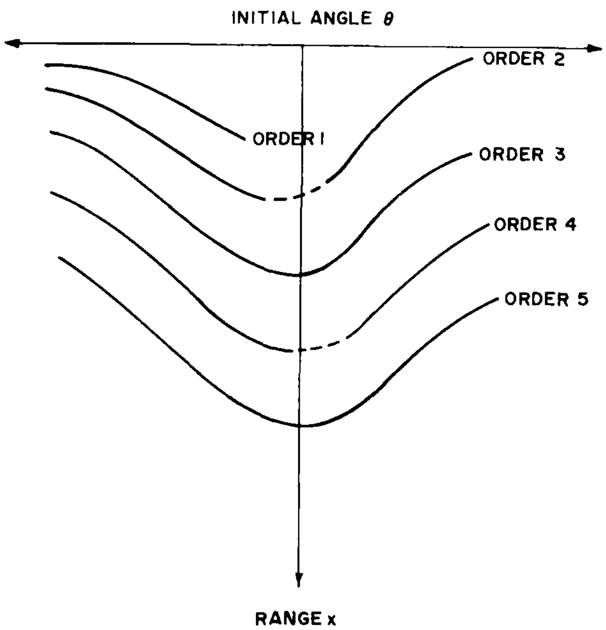

Fig. 4 — Order contours

For the reverberation from a given boundary, only those contours corresponding to that boundary are pertinent; they are odd for the surface, and even for the bottom. For example, to determine all ray paths between the source-receiver and the bottom at range x, consider a horizontal slice at range x, as shown in Fig. 5. We denote the three paths having initial angles of θ_1, θ_2, θ_3, by 1, 2, 3, respectively. Then the index set $m(x)$ of Eq. (10) consists of the integers 1, 2, and 3. Therefore, there are 3^2 or 9 possible routes from the source-receiver to the bottom at range x and back again.

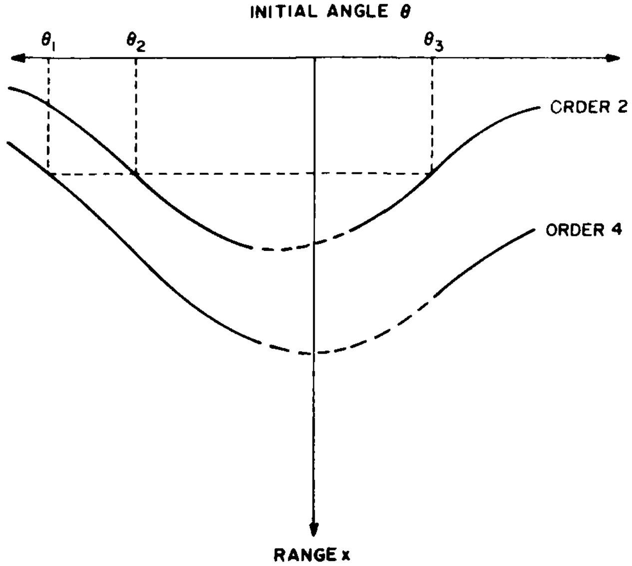

Fig. 5 — Calculation of launch angles, i.e., rays that meet bottom boundary (curves of even order) at range x

In practice a limited number of rays are traced, or, in other words, a finite number of points on the order contours are computed. Figure 6 illustrates the construction of two order contours of the type produced by the computer program. Here the *B*s represent the bottom hits and the *V*s represent refracted turning points or valleys determined by tracing rays with initial angles θ_1, θ_2, $\ldots \theta_n$. The order contours are approximated by linearly connecting consecutive computed bottom hits of the same order. For contour points between computed points, travel time, transmission loss, and the angle at which a ray encounters a boundary will be found by interpolating the corresponding computed values linearly with respect to range. The use of ray order thus enables the determination of all the ray paths connecting $S-R$ with a boundary element at any range x.

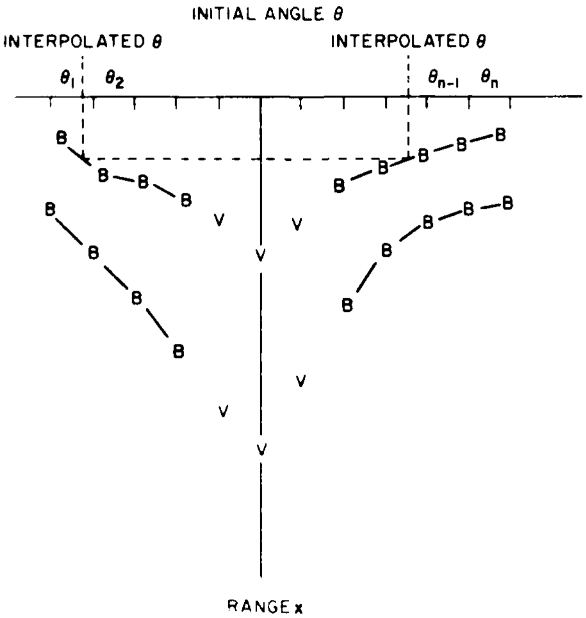

Fig. 6 — Computer construction of order contours

The order contours shown in Fig. 6 represent the kind of bottom contours obtained for a typical deep ocean sound-speed profile. The points of maximum range on each bottom order contour correspond to rays that just graze the bottom.

Figure 7 illustrates a smoothed surface order contour obtained for a typical deep ocean sound-speed profile. The two rays that just graze the bottom (corresponding to upward and downward initial angles) at ranges X_B and X_B' determine points B and B' on the contour. The two rays that just graze the surface at ranges X_A and X_A' occur at shallower initial angles. These rays contribute points A and A' on the contour. Thus rays corresponding to initial angles in the intervals $(\theta_{B'}, \theta_{A'})$ and (θ_A, θ_B) hit the surface but not the bottom. Rays providing the significant contributions to surface reverberation at long ranges have initial angles in the intervals defined above. Rays having steeper initial angles hit the bottom, thereby suffering bottom loss. The cumulative loss associated with several bottom hits causes these rays to have less effect on surface reverberation at long ranges where the higher order contours are relevant.

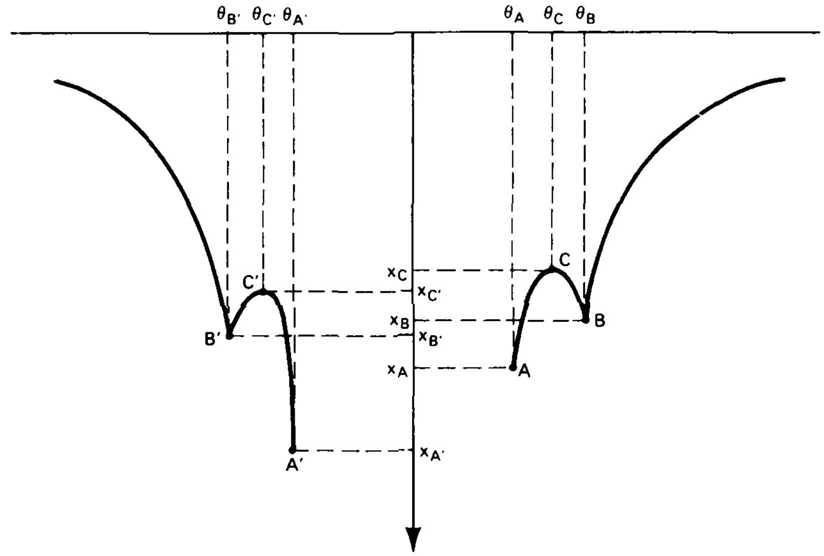

Fig. 7 — Smoothed surface order contour illustrating caustic behavior

The bistatic and pseudo monostatic cases require two ray traces, one for the source and one for the receiver. Each boundary point now corresponds to two distinct distances, source-to-boundary point distance and boundary point-to-receiver distance. Thus, for each boundary point, the values assumed by index i of Eq. (15) are determined by the properties of the source ray trace while the values assumed by index j of Eq. (15) depend upon the receiver ray trace. Two sets of order contours are required for each boundary (surface and bottom).

Transmission-Loss Calculations — Initially, an estimate of the ray intensity is made at each location on the contour determined by the ray tracing. Transmission loss consists of bottom loss, volume absorption, and geometric-spreading loss. Bottom loss is computed during ray tracing; whereas, the remaining losses are calculated after the order contours are constructed. Volume absorption is estimated by using Thorp's equation [4]. Geometric-spreading loss under our assumption of azimuthal symmetry is given by [5]:

$$L(x) = \left| \frac{x}{a^2} \frac{\sin\gamma}{\cos\theta} \frac{\partial x}{\partial\theta} \right|, \tag{17}$$

where a = unit reference distance, x = horizontal range, θ = initial source angle of ray, and γ=angle the ray makes with the horizontal at range x.

The derivative $\partial x/\partial\theta$ in Eq. (17) is approximated numerically at each point (x_i, θ_i) on a contour. If $\theta_{i-1}, \theta_i, \theta_{i+1}$ are successive initial angles of rays that are traced on this contour and x_{i-1}, x_i, x_{i+1} are the corresponding ranges, then

$$\frac{\Delta x_i}{\Delta\theta_i} = \frac{\Delta R_i}{\Delta R} \frac{\Delta R_{i-1}}{\theta_i - \theta_{i-1}} + \frac{\Delta R_{i-1}}{\Delta R} \frac{\Delta R_i}{\theta_{i+1} - \theta_i}, \tag{18}$$

where

$$\Delta R_{i-1} = |x_i - x_{i-1}|, \tag{19}$$
$$\Delta R_i = |x_{i+1} - x_i|, \text{ and}$$
$$\Delta R = \Delta R_{i-1} + \Delta R_i,$$

are used to estimate $|\partial x/\partial\theta|$. Transmission losses between computed boundary encounters will be found by interpolating linearly in range between the estimates at the computed boundary hits.

In using a ray approach to the problem of sound propagation, a serious deficiency arises when attempting to calculate geometric-spreading loss in near-caustic regions, that is, near locations where adjacent ray paths cross. In this region geometric spreading tends to zero, erroneously implying infinite intensity at the point of intersection. This phenomenon occurs when the derivative $\partial x/\partial\theta$ is equal to zero, or, as related to order contours, where the contour of order n, $x = f_n(\theta)$, has a stationary value. The surface order contour shown in Fig. 7 has caustics at points C and C'.

Also, as rays are traced to longer ranges from the source, the corresponding order contours tend to become irregular, owing to accumulated computation errors and numerical approximations. As a result, computed transmission loss for the higher-order contours becomes increasingly erratic and uncertain.

To handle these problems, a statistical ray-averaging procedure has been developed that operates on the order contour and effectively smooths irregularities in the propagation-loss predictions. It involves unfolding the order contour and providing an avenue around zeros in $\partial x/\partial\theta$. The procedure leads to an expression that is similar to the result obtained by Hardy [5]. Whereas Hardy's results call for ray intensity averaging as a function of depth, the method used in program CONTUR, developed by Palmer [6], performs ray intensity averaging as a function of range.

The presence of caustics corresponding to rays having initial angles within the intervals defined by the surface grazing ray angles and the bottom grazing ray angles implies that, for surface reverberation, the greatest variations in ray intensity occur in those intervals. Thus the initial angles of rays that contribute to the corresponding significant portions of the order contours must be sampled densely enough during ray-tracing to ensure accurate intensity calculations.

The total transmission loss at the surface or bottom boundary as a function of horizontal range from the source or receiver can be computed by using program TOTLOS. The program produces a Calcomp plot of transmission loss *vs* range.

Reverberation Calculation

Acoustic reverberation from either the ocean surface or bottom is calculated in program BISTRV. The calculation is performed by numerically evaluating Eq. (15). Program BISTRV requires two data files as input, each created by an execution of program CONTUR. The two files correspond to ray information from the source and from the receiver with respect to either the surface or bottom; i.e., one execution of program BISTRV calculates reverberation from either the ocean surface or ocean bottom.

The choice of a coordinate system for the evaluation of Eq. (15) depends in part on the underlying ray-trace programs. Since the ray-trace information such as transmission loss and travel time is ordered by range from the source or receiver, it is advantageous to use a coordinate system (p, r) in which p is the horizontal range along the propagation path out from the source, and r is the horizontal range along the return path back to the receiver. The ordered pair (p, r) constitutes a biradial coordinate system. The transformation of Eq. (15) to the biradial coordinate system yields (see the appendix):

$$R(t) = 2I_1 \iint_K \sum_{i \in m(p)} \sum_{j \in n(r)} C_{A_{ij}(t)}(p,r) \frac{B(\theta_i, \phi_i)\,\bar{B}(\theta_j, \phi_j)}{L_i(p)\,L_j(r)} \times \sigma(\theta_i', \phi_i, \theta_j', \phi_j) \frac{2pr\,dr\,dp}{[8h^2(p^2+r^2) - (p^2-r^2)^2 - 16h^4]^{1/2}}, \tag{20}$$

where $2h$ is the source-receiver horizontal separation. The source-receiver geometry and the region of integration are shown in Fig. 8.

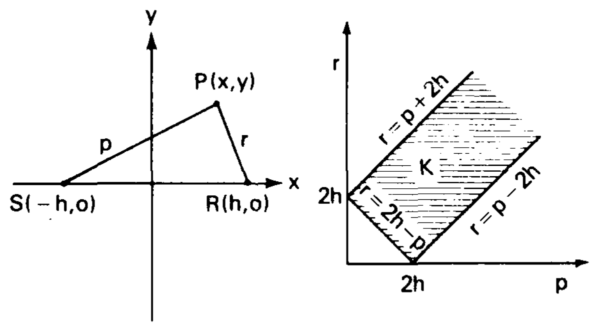

Fig. 8 — Source-receiver boundary plane geometry for bistatic situation in terms of both Cartesian and biradial coordinate systems

Equation (20) is evaluated numerically by incrementing the ranges p and r in equal step sizes from the source and receiver respectively. At present, the program assumes that the horizontal component of the beam patterns of both source and receiver are omnidirectional. This decreases the computation time significantly. Equation (20) now becomes

$$R(t_n) = I_1 \sum_k \sum_l \Bigg\{ \sum_i \sum_j C_{A_{ij}(t_n)}(p_k, r_l) \frac{b(\theta_i)\,\bar{b}(\theta_j)}{L_i(p_k)\,L_j(r_l)} \sigma(\theta_i', \phi_i, \theta_j', \phi_j) \times \frac{4p_k r_l \,\Delta p\, \Delta r}{[8h^2\,(p_k^2 + r_l^2) - (r_l^2 - p_k^2)^2 - 16h^4]^{1/2}} \Bigg\}, \quad n = 0, 1, \ldots, N. \tag{21}$$

The value $R(t_n)$ is defined to be the reverberation density per Δt occurring in the time interval $[t_n, t_{n+1}]$, where $\Delta t = t_{n+1} - t_n$, is the time increment. The values t_0, t_N and Δt are input parameters of program BISTRV. Values of the various ray parameters at each boundary point (p_k, r_l) are obtained by linearly interpolating between points on the appropriate order contours.

The scattering strength σ is approximated by using a backscattering table that is input by the user. This table defines backscattering as a function of grazing angle. The scattering value used for σ is that value in the table that corresponds to the average of the grazing angles θ_i' and θ_j' of the rays along paths i and j at the boundary point (p_k, r_l). The scattering strength σ is assumed to be independent of the azimuthal angles ϕ_i and ϕ_i.

Each term of the double summation in braces (i, j summation in Eq. (21)) represents the contribution to reverberation due to a particular reverberant path i, j from source to boundary area (defined by (p_k, r_l)) to receiver. As each contribution is calculated, the time interval over which it will be received is computed. The contribution is then added to those cells of the time array $R(t_n)$, $n = 0, 1, \ldots, N$, for those time values t_n falling within the computed time interval of reception. Thus each calculated contribution is added to one or more of the cells of array $R(t_n)$ (provided of course that the time interval of reception does not lie outside of the time interval of interest $[t_o, t_N + \Delta t]$). If the time interval of reception of a contribution only partially spans a time interval $[t_n, t_n + \Delta t]$ of the $R(t_n)$ array, then the contribution is reduced by that fraction of Δt before being added to that cell. After all the reverberation contributions have been calculated, each value of the $R(t_n)$ array is divided by Δt to obtain reverberation density per unit time.

The above calculation is valid for the true bistatic case where the source and receiver are separated in range, only if the environment is range independent; i.e., the ocean bottom is flat and one sound-speed profile is defined. This is because of the use of two-dimensional (range-*vs*-depth) ray tracing and assumed azimuthal symmetry of acoustic propagation from both the source and receiver. When the source and receiver are not coincident in range, concurrent azimuthal symmetry from the source and receiver is possible only in a range independent environment. However, when the source and receiver coincide in range, the environment may be range dependent.

Program BISTRV is also capable of providing the vertical distribution of reverberation at specific vertical angles and times input by the user. The angles at which reverberation values are provided are with respect to the receiver. The reverberation contribution at angle θ at time t is that portion of the total reverberation density corresponding to rays having acceptance angles in the angle interval of size $\Delta\theta$ (input) centered at θ, divided by $\Delta\theta$. This yields reverberation density per unit time per unit angle.

A condensed form of Eq. (21) is evaluated for pseudo monostatic and monostatic reverberation (see description of program BISTRV). The calculation assumes full 360° azimuthal omnidirectionality. However, the user can provide for horizontal beam widths by adjusting the value of source level input in program AVREVB which is discussed in the next paragraph.

The reverberation values are output on a file which is used by program AVREVB. This program plots a curve of reverberation *vs* time. Values of source level (possibly adjusted for azimuthal beam width for a monostatic calculation) and noise level are input parameters of program AVREVB. The program is also capable of averaging several reverberation curves and of time averaging.

DESCRIPTION OF PROGRAMS

The computer programs described in this report are currently operational on the Texas Instruments Advanced Scientific Computer (ASC) at the Naval Research Laboratory. This section describes the purpose and usage of each program involved in the sequence of calculations and plottings.

Program Name: PROFIL

Purpose: Calculate velocity gradients and curvatures as functions of depth from input velocity profiles. Create an output file containing the velocity profiles, their calculated derivatives, and a description of the bottom topography.

Usage:

Input

1. Punched Cards — Format

Card #1 TITLE (20A4)
TITLE — 80 column title for printed output.

Card #2 NECUR (I5)
NECUR — flag for earth's curvature correction.
= 1 if curvature correction is desired,
= 0 otherwise.

Card #3 ND1, NMO, NDY, NYR, NPRF (5I5)
ND1 — arbitrary identification number.
NMO — month
NDY — day } arbitrary date.
NYR — year
NPRF — number of velocity profiles to be input (NPRF$\geq$2).

Card #4 NBPTS, BOTMIN, BOTMAX, RMAX (I5,F15.4,2F10.4)
NBPTS — number of bottom points used to describe a linearly segmented bottom (2 $\leq$NBPTS $\leq$ 500).
BOTMIN — miminum bottom depth (m).
BOTMAX — maximum bottom depth (m).
RMAX — maximum range (nmi) of bottom points.

Type #5 (BTRNGE(I), BTDPTH(I), I = 1, NBPTS) (2F10.4)
BTRNGE — range (nmi) of bottom point.
BTDPTH — depth (m) of bottom.
Note: NBPTS cards of type #5 are read.

Type #6 R (F10.1)
R — horizontal range (nmi) at which the velocity profile to be read in is defined.

Type #7 (D(I), V(I), I = 1,M) (10F8.1)
D — depth (m).
V — velocity (m/s).
Note: Type #7 is repeated until M nonzero velocities are read in at a rate of up to 5 per card. (M$\leq$500)

1. Punched Cards — Format

Type #8 — 1 in columns 14 and 15 (I15)
signals that there are no more points in this velocity profile.

Note: Types #6-8 are repeated until NPRF profiles are read in. For a single profile environment the profile is read in twice, once at zero range and once at maximum range.

Type #9 — 7 in columns 14 and 15 (I15)
signals that there are no more profiles to be input.

Subroutines Called: DERIV, PARAM

Output

1. Printed: Velocity profiles with gradients and curvatures.
2. File: FT01F001. Catalogue this file for use as input for programs REVRAP and PLTENV.

Program Name: PLTENV

Purpose: Produce a Calcomp plot of velocity profiles and bottom topography from a file created by program PROFIL.

Usage:

Input

1. Punched Cards — Format

Card #1 RM, DM, XAXIS, YAXIS, VTR (5F10.3)
- RM — maximum range (nmi) of plot.
- DM — maximum depth (m) of plot.
- XAXIS — length of x-axis of plot in inches.
- YAXIS — length of y-axis of plot in inches.
- VTR — velocity to range scale for plotting. VTR equals the length in inches that represents 10 m/s for the velocity profile curves.

2. Files: Assign output file from program PROFIL to FT01F001.

Subroutines Called: LABEL, PLOTAX, PLOTPT

Output

Plot: A YAXIS by XAXIS Calcomp plot is generated. The plot includes bottom depth (m) versus range (nmi) and the velocity (m/s) vs depth (m) profiles located at their corresponding ranges (nmi).

Program Name: REVRAP

Purpose: Trace rays using iterative technique. Calculate accumulative bottom loss and part of the geometric spreading loss formula at each boundary hit or refractive reversal. Calculate travel time and horizontal range along with either the grazing angle for a boundary hit or the reversal depth for a ray reversal caused solely by refraction. Obtain the maximum surface grazing angle for each ray. Generate output file containing ray information at reversal points (refractions and boundary hits).

Usage

Input

1. Punched Cards — Format

Card #1 TITCAR — (20A4)

TITCAR — 80 column title for printed output.

Card #2 RNGINI, DPINIT, DELMAX, DELMIN, VFAEPS, DELSMX, SURFDE, TIMEMA — (8F10.3)

RNGINI — range of source (nmi) (usually zero).
DPINIT — depth of source (m).
DELMAX — first trial step size (m) used in predicting a new ray path position (e.g., 1000 m).
DELMIN — minimum allowable step size (e.g., 20 m).
VFAEPS — velocity field accuracy (e.g., 0.2 m/s).
DELSMX — maximum allowable change in the sine of the ray angle (angle from horizontal) between predicted positions (e.g., 0.02).
SURFDE — vertical distance (m) from surface or bottom within which a ray is to be considered as hitting that boundary (e.g., 0.05 m).
TIMEMA — travel time (s) after which a ray will be terminated.

Card #3 TLOSSM — (F10.3)

TLOSSM — intermediate transmission loss value (dB) after which a ray will be terminated (e.g., 200 dB).

Card #4 NGRPS, NRGRPS, KTAPE, MAXORD — (4I5)

NGRPS — number of cards of type #5 to be read in to define all the initial angles of the rays to be traced.

NRGRPS — rectification range identifier.
if positive — number of cards of type #6 to be read in to define "rectification" ranges.

if negative — negative of number of rectification ranges to be read in on cards of type #7 at a rate of 8 ranges per card.

1. Punched Cards — Format

Note: As each rectification range is reached, ray information obtained up to that range is printed and written on an output file. Any range at which a velocity profile has been defined by program PROFIL is considered an additional rectification range to those defined by card types #6 or #7.

KTAPE — file identifier
if positive — logical unit of output file if desired
(KTAPE $\geqslant$ 2 but $\neq$ 5, 6, or 7).
if nonpositive — no output file is generated.

Note: Output file will be assigned to FTONF001 where N = KTAPE.

MAXORD — maximum allowable boundary encounter order to which each ray is traced.

Type #5 ANGINC, ANGINI, ANGFIN (3F10.3)

ANGINC — angle increment (degrees) of group.
ANGINI — initial angle (degrees) in group.
ANGFIN — final angle (degrees) in group.

Note: The positive Z axis is directed downward; i.e., positive angles describe rays whose initial direction is towards the bottom. NGRPS cards of type #5 are input with two requirements:

1. Angles must be defined in increasing order from negative angles to positive angles,

2. An angle can be defined in at most one group, i.e., no overlapping by angle groups is permitted.

The maximum number of angles allowed is 1000.

Type #6 RGINC, RGI, RGF (3F10.3)

RGINC — rectification range increment (nmi).
RGI — initial rectification range (nmi).
RGF — final rectification range (nmi).

Note: NRGRPS cards of type #6 are input only if NRGRPS is positive. The first rectification range defined should be zero. A rectification range can be defined by at most one card of type #6, i.e., no overlapping. A maximum of 8000 total reversals are allowed in each rectification interval.

Type #7 (STRRNG(I), I = 1, NRGRPS) (8F10.3)

STRRNG(I) - Ith rectification range (nmi). (NRGRPS $\leqslant$ 300)

1. Punched Cards | Format

Note: (-NRGRPS) ranges are input at a rate of 8 per card only if NRGRPS is negative. The first rectification range defined should be zero.

Card #8 NINT, NTAB, (NINTAB(I), I=1, NTAB) (6I5)

NINT — number of range intervals for which different bottom loss tables are desired (1) ≤ NINT≤10).

NTAB — number of distinct bottom loss tables (1≤NTAB≤4).

NINTAB(I) — number of loss values in Ith table (NINTAB(I≤100).

Type #9 GRANG(I,J), DBLOSS(I,J) (2F10.3)

GRANG— grazing angle (degrees) (0 ≤ GRANG ≤ 90).
DBLOSS— corresponding bottom loss in positive dB.

Note: First NINTAB(1) cards of type #9 are input to define loss table #1, then NINTAB(2) cards of type #9 are input to define table #2, etc., until NTAB bottom loss tables are input.

Type #10 KTAB(I), ROFTAB(I) (I2,F8.1)

KTAB(I) — bottom loss table number to be used in range interval I (1 ≤ I ≤ NINT). Table number is determined by the order in which tables are read in.

ROFTAB(I) — maximum range (nmi) of Ith range interval.

Note: NINT cards of type #10 are read, i.e., one for each range interval the Ith range interval is given by $I_i = (R_{i-1}, R_i)$, $1 \leq i \leq$ NINT, where $R_0 = 0$ and $R_i = ROFTAB(I+1)$.

2. Files: Assign output file from program PROFIL to FT01F001.

Subroutines Called — HITBOT, HITSUR, ITRAT, RECORD

Output

1. Printed:
   a. Listing of input parameters.

   b. A table containing ray information at all reversal points (crests, valleys, surface hits, and bottom hits). The information is printed out for increasing initial angle (angle at source) for each retification interval. A description of the nine columns of the table is as follows:

Label	Description
REVERSAL	A running total of the number of reversals encountered by the program.
ANGLE	Ray angle in degrees at the source
CODE	An eight-digit hexidecimal word interpreted as three distinct numbers, AAABBBBC where AAA is the order of the reversal of the ray being described, BBBB is the ray number which is in one-to-one correspondence with initial angle, C is the type of ray reversal where 1 — surface hit 2 — bottom hit 3 — crest 4 — valley
RANGE (nmi)	Horizontal range of the reversal point in nautical miles.
GRZ ANG OR DEPTH	Grazing angle of the ray in degrees at the surface or bottom or the depth in meters of the ray at its reversal point.
TIME	Travel time in seconds for the ray to reach the reversal point.
TR LOSS	Intermediate transmission loss value (intensity units) obtained by evaluating the transmission loss formula (Eq.(17)) without including the $\partial x/\partial \theta$ factor. This factor is calculated in program CONTUR.
DB	Same transmission loss value in dB units.
MAX SURFACE GRZ	The maximum grazing angle with which the ray has encountered the surface. A value of 1000 indicates that the ray has not yet reflected from the surface.

2. File: FTONF001 where N = KTAPE (Card #4). Catalogue this file for use as input file for programs PLTCON, CONTUR, and possibly program MERGE.

Note: Rays are traced one at a time from the source to the first rectification range (input ranges plus ranges of sound speed profiles). Next reversal information is printed and written onto the output file. The process is repeated to each rectifica. n range.

Program Name: MERGE

Purpose: Merge two or more ray tracing files created by program REVRAP into a new single ray-tracing file which can then be used as input for programs PLTCON and CONTUR. Program MERGE enables the user to increase the number of rays traced from those originally traced by REVRAP by executing REVRAP again. The additional executions of REVRAP are used to trace rays with initial angles not traced by previous REVRAP executions. This eliminates the need to trace any ray more than once.

Usage

Input

1. Punched Cards — Format

Card #1 KTAPE, NOTAPE, NTAPE (3I5)

- KTAPE — logical unit number of main file.
- NOTAPE — number of input files to be merged (each file is the output file of a REVRAP execution).
- NTAPE — logical unit number of merged output file.

Type #2 MTAPE, NUMIN (2I5)

- MTAPE — logical unit number of a file to be merged (MTAPE $\neq$5,6,7).
- NUMIN — the number of rays from MTAPE file to be merged with those rays present on file KTAPE.

Type #3 KAY(I), I=1, NUMIN (16I5)

- KAY — Ray numbers of rays to be merged with rays present on the main file (KTAPE). These numbers are identified by examining the hexidecimal words labeled CODE in the printed output of that REVRAP execution that created the file now assigned to logical unit number MTAPE. Use the decimal equivalent of the ray number as defined in CODE (i.e., do not input the ray number as a hexidecimal number). Ray numbers must be in increasing order, i.e., KAY(I+1) > KAY(I).

Note: For each file to be merged, one card of type #2 followed by one or more cards of type #3 are required. It is not necessary to merge every ray traced by an auxiliary REVRAP execution. Instead type #3 cards enable the user to merg only those rays that are wanted. Of course, the new file, NTAPE, will contain all rays that are present on the original file KTAPE.

2. Files: Assign output file from program REVRAP which is interpreted as main file to FT0KF001, where K = KTAPE.

   Assign auxiliary output files created by additional executions of program REVRAP to FT0MF001, where M = MTAPE. Use a different value of MTAPE for each auxiliary file. (M $\neq$ 5, 6, 7).

Subroutines Called: There are no subroutines called.

Output

1. Printed: Information describing the execution of program MERGE is provided including the angles for which ray information exists on the new (output) file.

2. File: FTONF001, where N = NTAPE. Catalogue this file for use as input file for programs PLTCON and CONTUR.

Program Name: PLTCON

Purpose: Produce Calcomp plots of surface and/or bottom order contours from the output file generated by program REVRAP.

Usage:

Input

1. Punched Cards — Format

Card #1 (KTITLE(I), I=3, 20) — (18A4)

KTITLE — 72 column title for plot. (The program defines the first word of the title as either SURFACE or BOTTOM).

Card #2 KTAPE, (MINORD(I), MAXORD(I), I=1, 2) — (5I5)

KTAPE	—	logical unit number of input file.
MINORD(1)	—	minimum order of surface contour to be plotted.
MAXORD(1)	—	maximum order of surface contour to be plotted (can be 0).
MINORD(2)	—	minimum order of bottom contour to be plotted.
MAXORD(2)	—	maximum order of bottom contour to be plotted (can be 0).

Card #3 AG1, AG2, YAXIS, RG1, RG2, XAXIS — (6F10.2)

AG1	—	minimum y-coordinate (initial angle — degree).
AG2	—	maximum y coordinate (final angle — degree).
YAXIS	—	length of y-axis in inches ($\leqslant$ 10 in.).
RG1	—	minimum x-coordinate (minimum range—nmi).
RG2	—	maximum x-coordinate (maximum range—nmi).
XAXIS	—	length of x-axis in inches.

2. Files: Assign output file from program REVRAP to file FTONF001 where N = KTAPE (card #2).

Note: Present dimensioning allows a maximum of 8000 ray reversals to be input.

Subroutines Called: ORDPLT, PLOTAX, PLOTPT

Output

1. Printed:
Order contours consisting of rays of the same order encountering a boundary are listed, first all surface contours then all bottom contours. Each contour is comprised of one or more monotonic segments, i.e., segments where the horizontal range of rays encountering the boundary either increases or decreases as a function of initial angle. For each order contour, rays are grouped by these monotonic segments and listed in the order of increasing horizontal range. The segments are ordered by increasing initial angle.

2. Plots:
Calcomp plots of order contours corresponding to surface and/or bottom boundary encounters are generated. The contours are plotted as functions of initial angle (y-axis) versus horizontal range (x-axis). Surface contours and bottom contours are plotted on separate sets of axes.

Program Name: CONTUR

Purpose: Read output file created by program REVRAP, create order contours and complete transmission loss calculations by computing two types of transmission loss. Create an output file containing ray information including one type of transmission loss calculation (corrected for source or receiver beam pattern). In contrast to program PLTCON, each execution of program CONTUR processes either surface or bottom boundary hits but not both.

Usage

Input

1. Punched Cards — Format

Card #1 TITLE — (20A4)
TITLE — title card for printed output.

Card #2 KORD, KFRQ, KTAPE, LTAPE, ITL — (5I5)

KORD — type of order contours to be processed
= 1 for surface contours
= 2 for bottom contours

KFRQ — frequency of source in Hz. (Used in Thorpe's equation.)
KTAPE — logical unit number of input file.
LTAPE — logical unit number of output file.
ITL — flag for type of transmission loss calculation to be output on file.

= 1 minimum of: a. statistical averaging of ray intensities, b. ray bundle estimates (i.e., value which predicts the larger transmission loss is chosen),

= 2 ray bundle estimates.

1. Punched Cards | Format

Type #3 (BP(I), I=1,181) (8F10.0)

BP — the vertical beam pattern of the source or receiver. A total of 181 values are read in by means of 23 input cards. The pattern is to be specified from -90° (towards surface) to 90° (towards bottom) in 1° steps. Values are in dB units with all values less than or equal to zero.

Note: If an Ominidirectional pattern with a value of zero dB at all angles is desired, then 1 card of type #3 with BP(1) = 99. may be read in instead of 23 blank cards.

2. Files: Assign output file from program REVRAP to file FTONF001 where N = KTAPE.

Note: Ray reversals are input in blocks corresponding to rectification intervals. (See note following the description of Card #4 in program REVRAP.) All ray reversals except those corresponing to type KORD (Card #2) are deleted from each block before the next block is input. The total number of ray reversals present in CONTUR is at one point as large as the number of ray reversals of type KORD determined previously plus the number of ray reversals in the last rectification range. This total may not exceed 8000 with present dimensioning.

Subroutines Called: FORMOC

Output

1. Printed:

a. Information based on input parameters

b. Order contours consisting of rays of the same order encountering the requested boundary are listed. The listings are similar to those found in the printed output of program PLTCON.

2. Files: FTONF001 where N = LTAPE (Card #2). Catalogue this file for use as input file for programs TOTLOS and BISTRV.

Program Name: TOTLOS

Purpose: Compute transmission loss (corrected for source or receiver beam pattern) at a boundary (surface or bottom) as a function of range. Produce a Calcomp plot of transmission loss *vs* range.

Usage:

Input

1. Punched Cards | Format

Card #1 (TITLE (K), K = 3,20) (18A4)

TITLE — 72 column title for plot. (The program defines the first word of the title as either SURFACE or BOTTOM.)

1. Punched Cards — Format

Card #2 KTAPE, MRG (2I5)

KTAPE — Logical unit number of input file.
MRG — maximum range (nmi) for which transmission loss is to be computed.

Card #3 XMIN, XMAX, YMIN, YMAX, XAXIS, YAXIS (6F10.5)

XMIN — minimum x-coordinate (initial range in nmi).
XMAX — maximum x-coordinate (final range in nmi).
YMIN — bottom y coordinate (maximum transmission loss in dB).
YMAX — top y coordinate (minimum transmission loss in dB).
XAXIS — length of x-axis in inches.
YAXIS — length of y-axis in inches ($\leq$ 10 in.).

2. File: Assign output file from CONTOUR to FTONF001 where N = KTAPE (Card #2).

Subroutines Called: PLOTAX, PLOTPT

Output

1. Printed:
Transmission loss in dB units as a function of horizontal range in steps of 0.5 nmi is listed in a two column table.

2. Plot:
A Calcomp plot of transmission loss *vs* range is generated.

Program Name: BISTRV

Purpose: Calculate either monostatic or bistatic reverberation as a function of time. Compute the vertical distribution of reverberation at up to 20 specified times. If the source and receiver are separated in horizontal range (bistatic case) then the calculation is correct only if sound velocity and bottom are range independent. Either surface or bottom reverberation is calculated in a single execution of program BISTRV.

Usage:

Input

1. Punched Cards — Format

Card #1 TITLE (20A4)
TITLE — 80 column title for printed output.

Card #2 MONO, MXSORD, MXRORD (3I5)

MONO — flag for type of calculation,
= 0 if a bistatic calculation is to be made,
= 1 if a monostatic calculation is to be made.
MXSORD — maximum order of contour to use from source file.
MXRORD — maximum order of contour to use from receiver file.

1. Punched Cards		Format
Card #3	RABSMA, PABSMA	(2F10.2)

RABSMA — maximum range from receiver (nmi) to consider in reverberation calculation.

PABSMA — maximum range from source (nmi) to consider in reverberation calculation.

Card #4	X1SOUR, X1REC, DXPR, SEP	(4F10.2)

X1SOUR — minimum p coordinate (source) of grid point (nmi).
X1REC — minimum r coordinate (receiver) of grid point (nmi).
DXPR — range increment for p and r coordinates.
SEP — source-receiver separation in horizontal range (nmi).

Note 1: For monostatic calculation, input variables XIREC and SEP are disregarded. They are defined by the program as explained below.

Note 2: The variables defined on Card #4 refer to a (p,r) coordinate system where p is the horizontal range from the source S to the scattering point P and r is the horizontal range from P to the receiver point R. The distances p,r and SEP are shown in Fig. 9. Here the $x-y$ plane is the plane containing the ocean boundary (surface or bottom), P is the scatter point, and $\hat{S}$ and $\hat{R}$ are projections of S and R onto the boundary plane. It is apparent that for a given $S-R$ separation of SEP, the following three inequalities must be satisfied:

$$p + r \geqslant SEP,$$

$$r + SEP \geqslant p,$$

$$p + SEP \geqslant r.$$

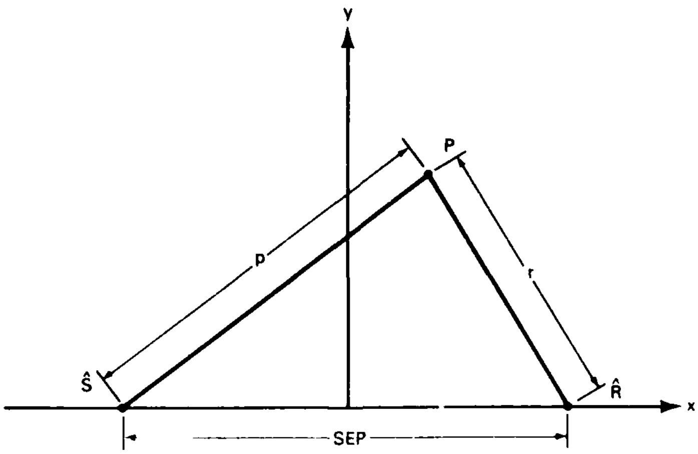

Fig. 9 — Notation and geometry of bistatic configuration for one scattering path

Figure 10 shows the scattering region in the $p-r$ plane along with other variables defined on Card #4.

There are values of certain parameters that will collapse the general reverberation calculation to the monostatic case. These are:

1. X1SOUR = X1REC (usually 0)

2. SEP = DXPR/2

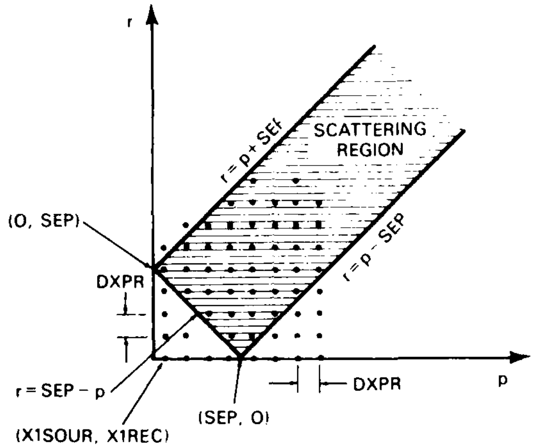

Fig. 10 — Bistatic scattering region in p-r plane

Under these conditions the (p, r) grid points are such that $p = r$ as shown in Fig. 11. The program automatically defines X1REC equal to X1SOUR and SEP equal to DXPR/2 for monstatic calculation (MONO = 1).

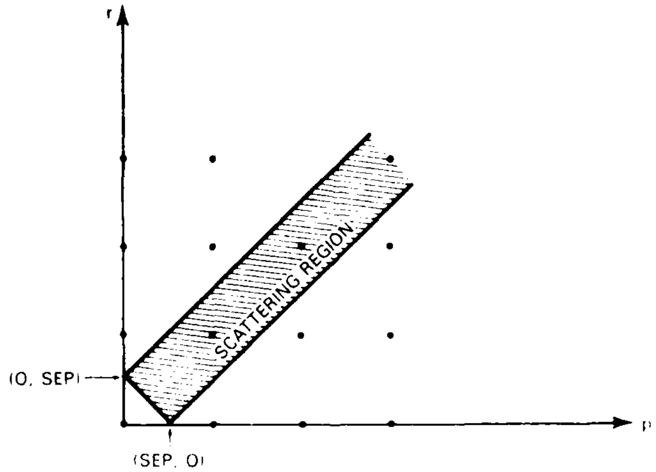

Fig. 11 — Monostatic scattering region in p-r plane

There are requirements which must be observed when calculating bistatic reverberation. For a given source receiver separation SEP, one must satisfy:

1. DXPR = SEP/n, where n is a positive integer,

2. |X1SOUR − X1REC| = (m + 1/2) (DXPR), where m is a nonnegative integer.

To illustrate, the actual selection of parameters for a bistatic calculation would probably occur in the following way:

Step 1: Select grid increment DXPR based upon the given separation SEP such that DXPR = SEP/n.

Step 2: Select X1SOUR = 0, X1REC = DXPR / 2 (or vice versa). The Jacobian of the transformation between $x-y$ coordinates and $p-r$ coordinates is not defined on the boundaries of the scattering region (see Fig. 10). A sufficient condition for grid points to miss the boundaries is that X1REC not be an integer multiple of DXPR. The further restriction that X1REC = DXPR/2 assures that the grid squares represent an area equal to the area of the integration region (except for very early arrivals).

For example choosing DXPR = SEP/4 (Step 1) along with X1REC = 0 and X1SOUR = DXPR/2 (Step 2) yields the grid shown in Fig. 10.

Steps 1 and 2 determine the grid to calculate reverberation for all times. If reverberation is desired only for scattering areas beyond certain ranges or for times greater than a minimum time, then X1SOUR and X1REC may be redefined as follows. In Step 2, select X1SOUR = m_1 (DXPR), X1REC = (m_2 + 1/2)(DXPR) (or vice versa) where m_1 and m_2 are integers chosen so that X1SOUR and X1REC are equal to the minimum ranges to be considered for the reverberation calculation.

If reverberation is desired for times greater than some minimum time t_o, then appropriate values of X1SOUR and X1REC, can be defined by taking the velocity of sound in the ocean to be approximately 0.8 nmi/s. Remember that the time at which reverberation is calculated is the two-way travel time from the source to the scattering point and then to the receiver. To illustrate this last point, suppose reverberation values are desired for times greater than 90 s, which corresponds to a total distance of approximately 72 nmi. One would then choose $m_1 = m_2$ such that X1SOUR and X1REC are somewhat less than 36 nmi (say 30 nmi), thus eliminating calculations for reverberation times not desired. A similar calculation would apply to the monostatic case.

Card #5 DUR (F10.2)

DUR — pulse duration of source (s).

1. Punched Cards | Format

Card #6 T1, DT, T2, THETA1, DTHETA, THETA2 (6F10.2)

T1 — initial time at which to compute reverberation (s)
DT — time increment (s)
T2 — final time at which to compute reverberation (s)
THETA1 — initial vertical angle (degree) at which to compute the vertical distribution of reverberation (-90° direction is toward surface).
DTHETA — angle increment (degree).
THETA2 — final vertical angle (degree) at which to compute the vertical distribution of reverberation.

Note 1: A maximum of 2000 times may be defined.
A maximum of 181 angles may be defined.
If the vertical distribution of reverberation is not to be calculated, the last three fields in this card may be left blank

Note 2: The reverberation at time t is defined to include all contributions to reverberation occurring in the time interval from t to $t + DT$. This value is then divided by DT so that the final value is actually a reverberation density per unit time.

Type #7 VERTTI(IV) (8F10.2)

VERTTI — the IVth time at which to compute the vertical distribution of reverberation. Up to 20 times are allowed with a zero value indicating the end of input times.

Note: At least one card is required (blank card indicates no vertical distribution calculation). The reverberation contribution at angle θ at time t is determined by that portion of the the total contribution to reverberation density (defined above) corresponding to rays having receiver vertical angles in the angle interval centered at θ and having angular size DTHETA. The value obtained is divided by DTHETA to yield reverberation density per unit time per unit angle.

Type #7 SIG (8F10.2)

SIG — An array of 91 values defining a backscattering table for the appropriate boundary as a function of grazing angle from 0° (tangent to boundary) to 90° (normal to boundary) in 1° steps. Values are in dB $\leqslant 0$.

2. Files: Assign source file generated by program CONTUR TO FTO1F001.

Assign receiver file generated by program CONTUR to FT02F001.

Subroutines Called: There are no subroutines called.

Output

1. Printed:
   a. Listing of input parameters.
   b. Table of reverberation in both dB units and intensity units *vs* time. The reverberation values are relative to an on axis source level of 0 dB.
   c. Tables of the vertical distribution of reverberation at the receiver *vs* angle for the times specified (if calculated).

2. Files:

FT10F001	Catalogue this file for use as input file for program AVREVB. The file contains the reverberation *vs* time results just as they are printed.
FT20F001	This file contains the vertical distributions of reverberation (if calculated).

Program Name: AVREVB

Purpose: Compute a weighted average of one or more reverberation envelopes calculated by program BISTRV. The resulting curve is time averaged, adjusted for source and noise levels, and plotted.

Usage:

Input — Format

1. Punched Cards

Card #1 TITLE (20A4)

TITLE — 80 column title for plot.

Card #2 NOCRVS, NOAVE, NOPTS (3I5)

NOCRVS — number of envelopes to be averaged.
NOAVE — number of discrete times that time averaging is performed over.
NOPTS — number of points in each envelope.

Card #3 XMIN, XMAX, YBOT, YTOP, XAXIS, YAXIS, ANL, SL (8F10.3)

XMIN — minimum time (s) used for labeling and scaling plot.
XMAX — maximum time (s) used for labeling and scaling plot.
YBOT — minimum reverberation level (dB) used for plotting.
YTOP — maximum reverberation level (dB) used for plotting.
XAXIS — length of x or time axis in inches.
YAXIS — length of y or reverberation level axis in inches ($\leqslant 10$)
ANL — noise level in dB re: source level.
SL — source level in dB re: 1 yard

1. Punched Cards — Format

Type #4 WT (F5.0)

WT — The weight assigned to the Nth envelope input, N = 1, 2, 3, ... , NOCRVS,

2. Files: Assign output files from BISTRV to FT01F00N where N = 1, 2, 3 . . . . NOCRVS. Thus BISTRV is run "NOCRVS" times, creating "NOCRVS" files each of which must be separately assigned before running AVREVB.

Subroutines Called: PLOTAX, PLOTPT

Output

1. Printed:
   a. Input variables except for plotting parameters.

   b. Reverberation values in dB both before and after being adjusted by source and noise levels and being time averaged.

2. Plot: A Calcomp plot of the adjusted reverberation vs time is generated.

EXAMPLES OF REVERBERATION CALCULATIONS

Two examples illustrating the use of the sequence of computer programs for predicting acoustic reverberation are presented. The first example is a "pseudo" monostatic reverberation calculation, i.e., a calculation where the acoustic source and receiver positions differ in depth but not in horizontal range. The second example is a bistatic reverberation calculation.

Figures 12 and 13 show how the computer programs are used to calculate reverberation. Input for program PROFIL consists solely of punched cards. Inputs for all other programs consist of punched cards and data files created by previously executed programs.

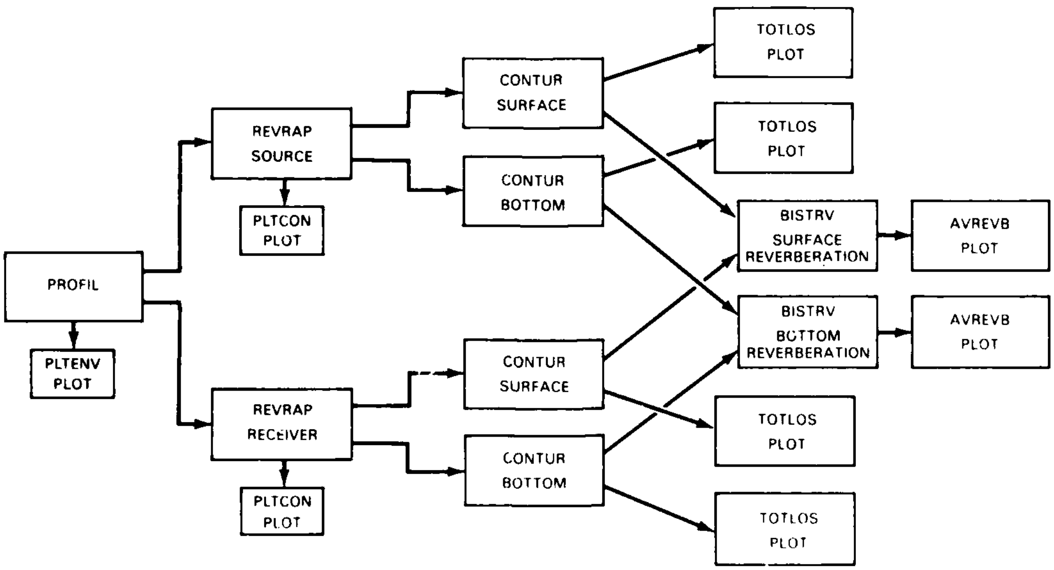

Fig. 12 — Implementation of ray-tracing and reverberation programs

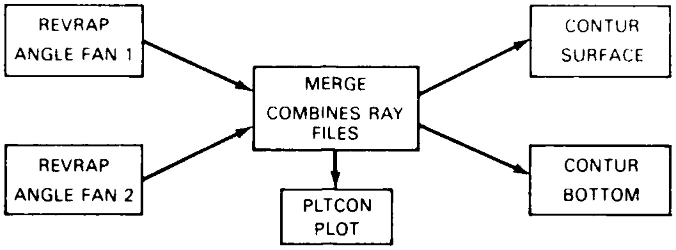

Fig. 13 — Implementation of MERGE program

The computer programs were compiled and executed on the Texas Instruments Advanced Scientific Computer (ASC) at the U. S. Naval Research Laboratory.

Example 1: "Pseudo" Monostatic Reverberation

Ocean Environment

Range independent sound speed field (North Atlantic winter profile used).

Linearly upward sloping ocean bottom with slope of 1m/km.

Bottom loss table and backscattering tables defined consistent with area of North A.. .itic corresponding to sound-speed profile

Acoustic Source Characteristics

Source depth: 400 m.

Source frequency: 200 Hz.

Source beam pattern: omnidirectional in azimuthal direction, possesses vertical directivity (see Fig. 15).

Pulse length: 10 s.

Source level: 200 dB.

Acoustic Receiver Characteristics

Receiver depth: 500 m.

Receiver beam pattern: omnidirectional both horizontally and vertically.

Calculations Desired

Acoustic reverberation from both the ocean surface and ocean bottom as a function of time up to 400 s after onset of source pulse.

Figure 14 shows the output plot of program PLTENV. Although the velocity profile is not shown extending to the line representing the ocean bottom, the profile is linearly extrapolated to the bottom for all ray tracing calculations.

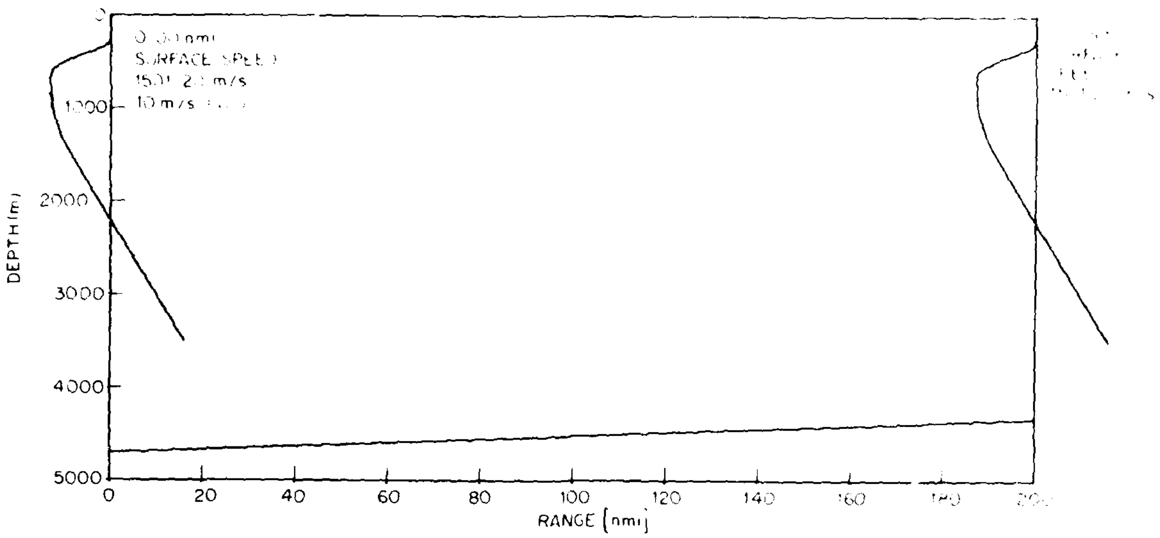

Fig. 14 — Output of PLTENV (Example 1)

Ray tracing calculations were performed by program REVRAP. The resulting rays were reordered in programs PLTCON and CONTUR to produce order contours. Surface and bottom order contours were formed and plotted by program PLTCON. Order contours for the source depth are shown in Figs. 15 and 16. Similar plots would be produced for the receiver depth.

The angle fans used in program REVRAP for the source and receiver were motivated by noting that the greatest variations in intensity at the boundaries and the major contributions to reverberation from the boundaries will occur for rays whose initial angles lie in an interval slightly larger than the interval defined by the surface grazing ray angle and the bottom grazing ray angle. Using Snell's law and extrapolating the input velocity profile (or the earth's curvature corrected profile printed by program PROFIL) to the bottom, the bottom grazing angles and the surface grazing angles were approximated. The actual values calculated for the source depth were an initial angle of 4.8° for grazing to the surface and initial angles of 13.7° at a bottom depth of 4700 m and 12.8° at a bottom depth of 4329.6 m (the depth at maximum range) for grazing to the bottom. From these estimates, the angles chosen for program REVRAP were: 1.0° steps from −45° to −20°, 0.5° steps from −19.5° to −15.0°, 0.2° steps from −14.8° to 0°, and similar increments for the positive angles.

Rays were traced to a maximum range of 200 nmi. If one assumes that acoustic waves in water travel at a rate of approximately 0.8 nmi/s then 200 nmi corresponds to a travel time of 250 s. The maximum two-way travel time from source to scattering point and back to receiver could, therefore, be as large as 500 s, which is well above the 400 s defined by the example.

Orders up to order 18 were requested in program REVRAP to ensure that all significant rays needed for intensity calculations were traced. The maximum reverberation time of 400 s corresponds to a source/receiver to scattering point range of about 160 nmi. Examination of Fig. 15 shows that the choice of 18 as a maximum order (17 for surface orders) was adequate.

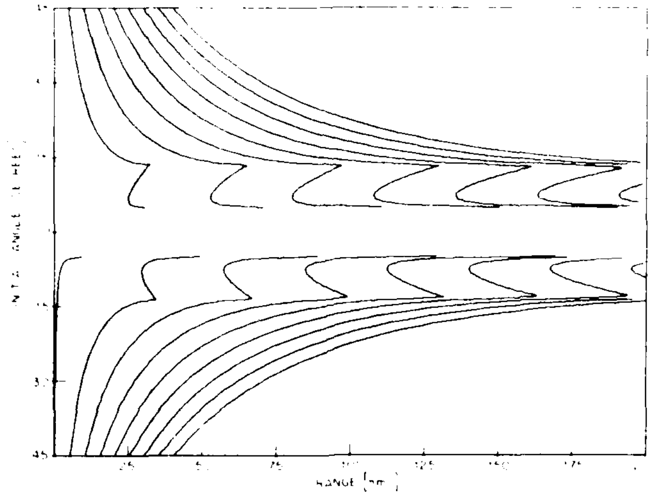

Fig. 15 — Order contours for source to surface (Example 1)

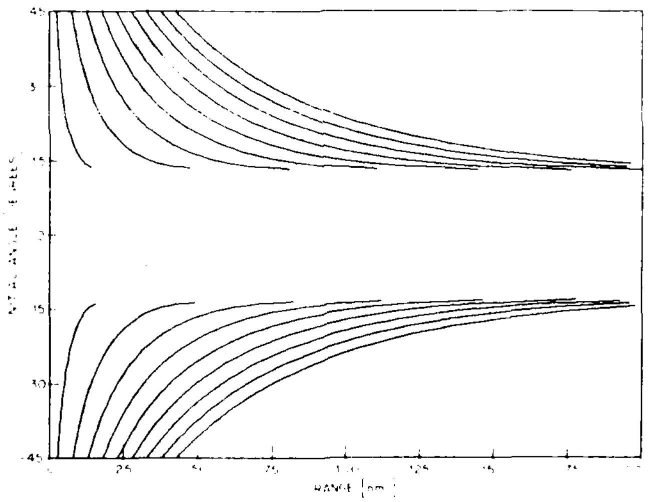

Fig. 16 — Order contours for source to bottom (Example 1)

The vertical beam pattern defined for the source is shown in Fig. 17. This pattern was input in program CONTUR by means of punched cards. Note that, since the ray angle fans in program REV-RAP were defined from −45° to 45°, values of the beam pattern outside that interval were not used in any computations; i.e., the angular limits of the vertical beam pattern are determined by the ray angle fan. Program CONTUR, which calculates final ray intensities, was executed four times, twice with the beam pattern defined for the source and twice with the omnidirectional pattern defined for the receiver (see Fig. 12). The two executions for each beam pattern produce first surface and then bottom order contours.

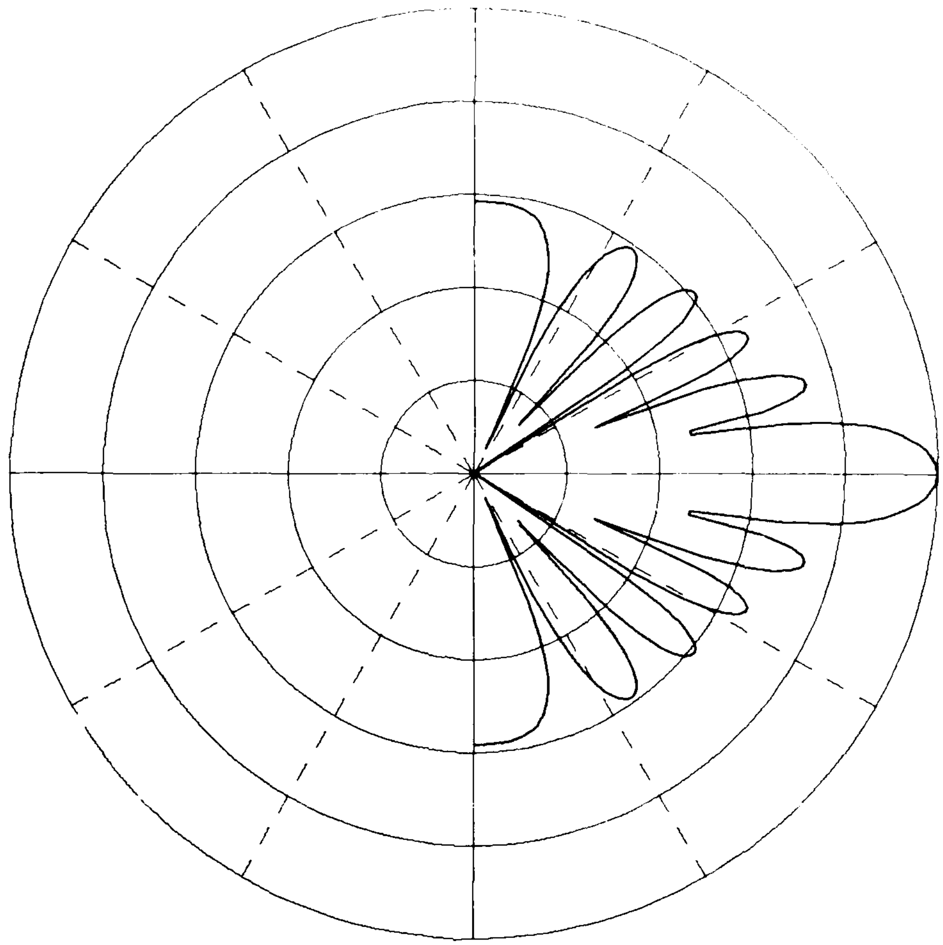

Fig. 17 — Source vertical beam pattern (both examples)

Results of four executions of program TOTLOS are shown in Figs. 18 through 21. The greater transmission loss for the source (400-m depth) is due to its vertical beam pattern, the receiver (500-m depth) being omnidirectional in the vertical.

Calculation of reverberation was accomplished by program BISTRV. Two executions were required, one for surface reverberation and one for bottom reverberation. Program AVREVB added the source level of 200 dB to each reverberation calculation and created a plot of reverberation *vs* time.

Surface and bottom reverberation curves are shown in Figs. 22 and 23. It is expected that regions of high reverberation generally correspond to regions of low transmission loss. This is indeed the case as can be observed by converting the reverberation arrival time axes of Figs. 22 and 23 to range from source/receiver, and then comparing Figs. 22 and 23 with Figs. 18 through 21.

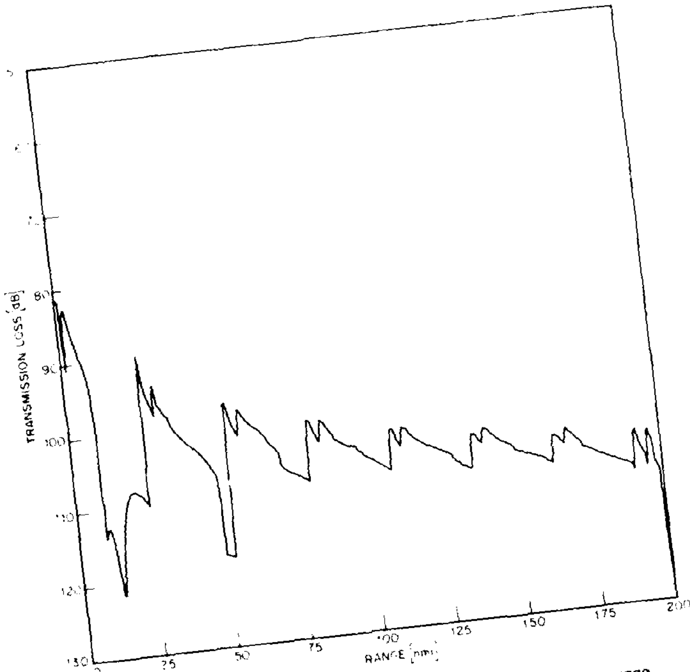

Fig. 18 — Transmission loss *vs* range at surface from source
(Example 1)

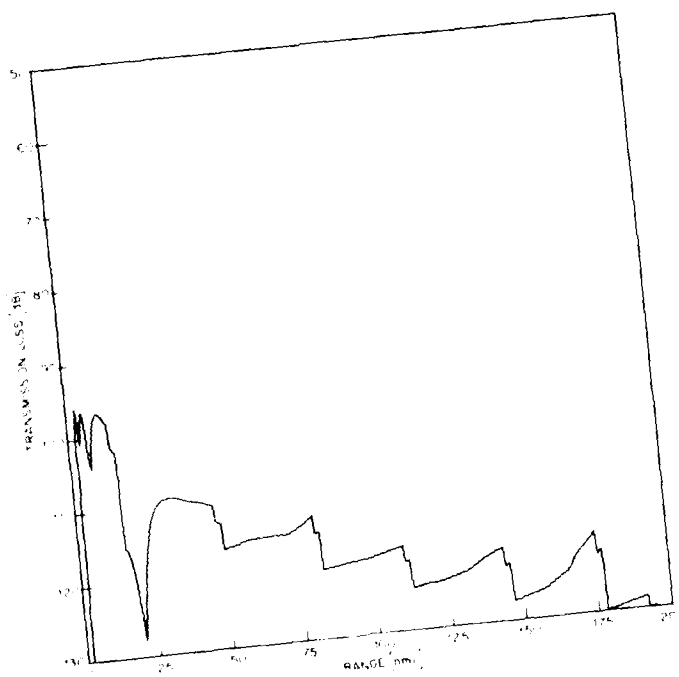

Fig. 19 — Transmission loss *vs* range at bottom from source
(Example 1)

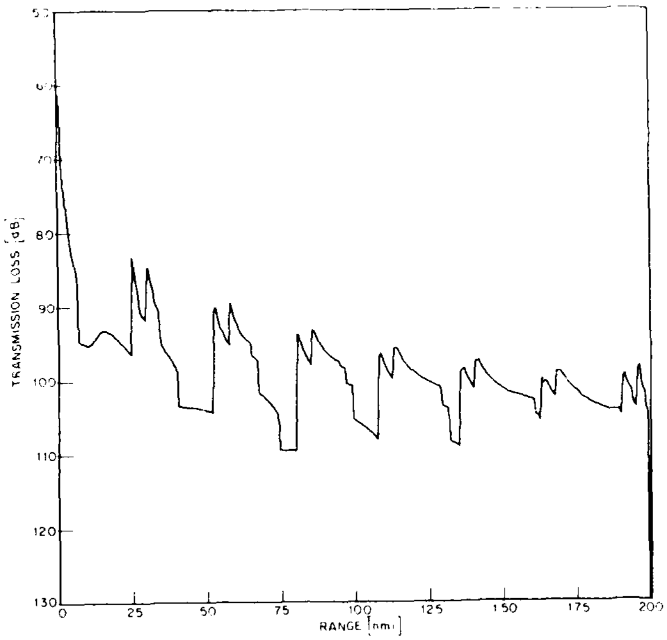

Fig. 20 — Transmission loss *vs* range at surface from receiver (Example 1)

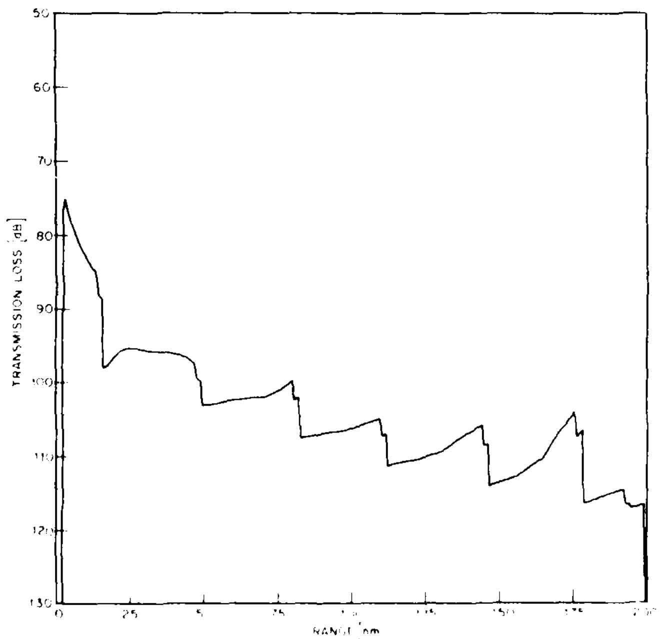

Fig. 21 — Transmission loss *vs* range at bottom from receiver (Example 1)

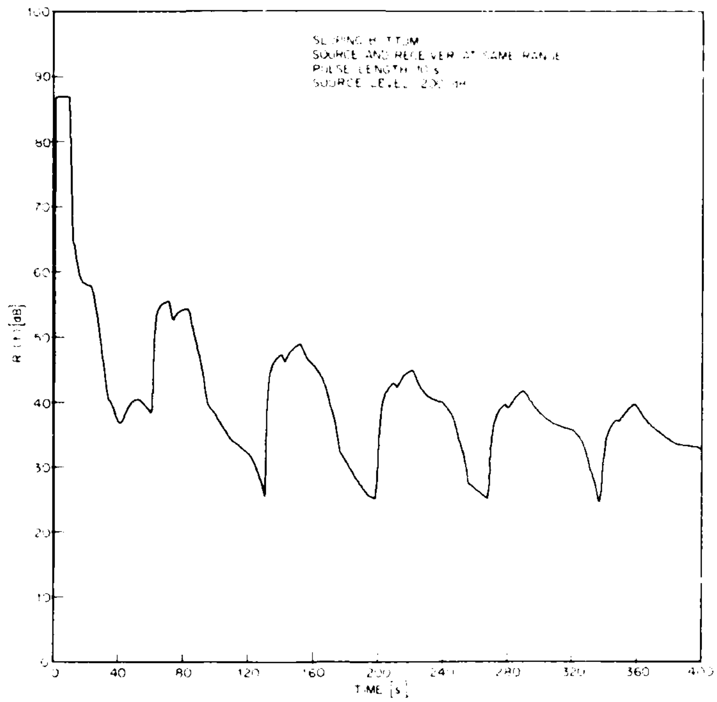

Fig. 22 — Reverberation envelope for surface interaction (Example 1)

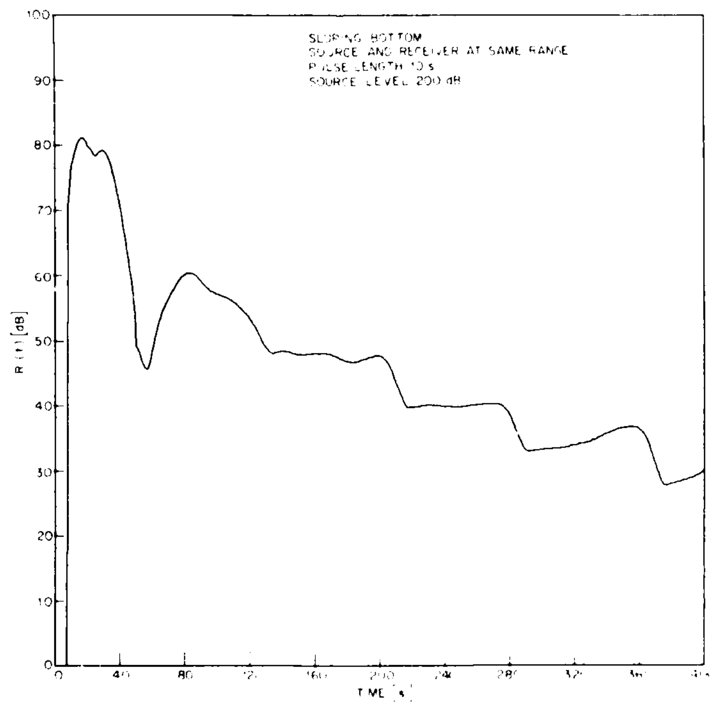

Fig. 23 — Reverberation envelope for bottom interaction (Example 1)

Example 2: Bistatic Reverberation

This example differs from Example 1 in that the ocean bottom is now defined as flat and the receiver is separated from the source horizontally by 40 nmi, thus defining a true bistatic situation.

Input parameters used for all computer program executions with the exception of programs PROFIL and BISTRV were exactly the same as those used in Example 1. The ocean bottom was defined to be of constant depth in program PROFIL and the 40-nmi source-receiver separation was input in program BISTRV.

Output plots of program AVREVB are shown in Figs. 24 and 25. The slight rippling of the curves is due to a grid size of 1.0 nmi being defined in BISTRV (variable DXPR) rather than a grid size of 0.2 nmi which was used in Example 1. The smaller grid size would have eliminated the ripples. However, the associated execution time for BISTRV would have increased more than tenfold over the actual execution time using the larger grid size.

Execution Time

The solution to each of the two examples required 18 separate program executions as shown in Fig. 12. The total time required to execute all necessary programs for Example 1 was approximately 200 s. A time of 125 s was required for the two executions of program REVRAP alone. Example 2 took about 570 s to run, 410 s of which were required to run program BISTRV twice. Table 2 shows typical execution times for each program.

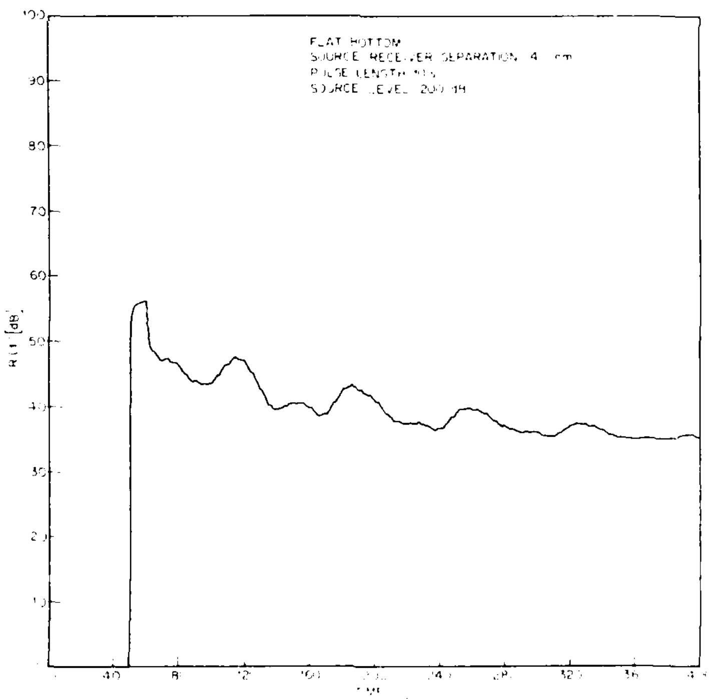

Fig. 24 — Bistatic reverberation envelope for surface interaction (Example 2)

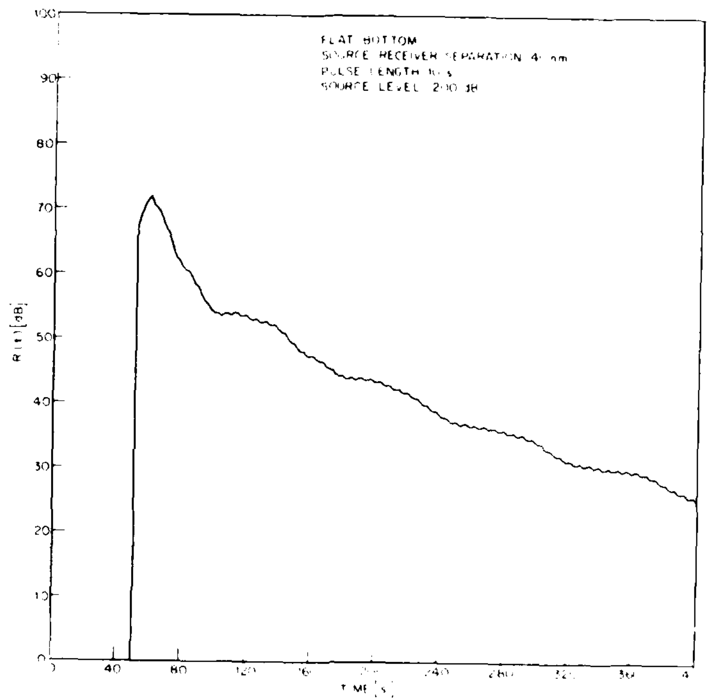

Fig. 25 — Bistatic reverberation envelope for bottom interaction (Example 2)

Table 2 — Execution Time for a Single Execution of Computer Programs used for Examples 1 and 2 using the Texas Instruments ASC Computer at NRL

Program Name	Typical Execution Time (s)
PROFIL	0.2
PLTENV	0.5
REVRAP	62
MERGE	—
PLTCON	5
CONTUR	6
TOTLOS	1
BISTRV (Example 2)	
Surface	250
Bottom	160
BISTRV (Example 1)	
Surface	23
Bottom	15
AVREVB	1

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Appendix
TRANSFORMATION TO BIRADIAL COORDINATES

We wish to transform Eq. (15),

$$R(t) = I_1 \int \int_A \sum_{i \in m(X,Y)} \sum_{j \in n(X,Y)} C_{A_{ij}(t)}(X,Y) \frac{B(\theta_i, \phi_i)\, \bar{B}(\theta_j, \phi_j)}{L_i(X,Y)\, L_j(X,Y)} \sigma(\theta_i', \phi_i, \theta_j', \phi_j)\, dX\, dY, \quad \text{(A1)}$$

into the biradial coordinate system. The individual symbols in Eq. (A1) are defined in the main text. For brevity we write

$$R(t) = I_1 \int \int_A f\, dX\, dY. \quad \text{(A2)}$$

Since we have assumed a flat bottom and a single sound-speed profile for the bistatic case, propagation characteristics are symmetric about the vertical plane containing the source and receiver. Also, in this report both the source and the receiver beam patterns are omnidirectional in the horizontal plane. Therefore f is symmetric about the line joining the (X, Y) coordinates of the source and receiver. This line is referred to as the source-receiver axis. Thus,

$$\int \int_A f\, dx\, dy = 2 \int \int_{A^+} f\, dx\, dy, \quad \text{(A3)}$$

where A^+ is the upper half plane of the boundary area A about the source-receiver axis. Clearly, a more general formulation where the beam patterns are not symmetric is possible, but for simplicity, we restrict our attention to the symmetric case.

If we refer back to Fig. 8, we can see that the Cartesian coordinates and the biradial coordinates are related by the equations

$$p^2 = Y^2 + (h+X)^2 \quad \text{(A4)}$$

and

$$r^2 = Y^2 + (h-X)^2. \quad \text{(A5)}$$

Solving for $X(p, r)$ and $Y(p, r)$, we obtain the transformation equations

$$X = \frac{p^2 - r^2}{4h} \quad \text{(A6)}$$

and

$$Y = \frac{[8h^2(r^2+p^2) - (r^2-p^2) - 16h^4]^{1/2}}{4h}. \quad \text{(A7)}$$

From Eqs. (A4) and (A5) it is clear that each point in A^+ has a unique image point in the (p, r) plane. What we seek now is the image of A^+ in the (p, r) plane. This is the set of (p, r) such that

$$[8h^2(r^2+p^2) - (r^2-p^2)^2 - 16h^4] \geqslant 0. \quad \text{(A8)}$$

Factoring inequality (A8) we obtain

$$(r - p + 2h)\ (r + p + 2h)\ (p - r + 2h)\ (r + p - 2h) \geqslant 0. \tag{A9}$$

An examination of this inequality shows the set of (p, r) we seek is the region K shaded in Fig. 8, and that the source-receiver axis $Y = 0$ folds into the boundary of K.

The transformation of Eq. (A2) into the biradial coordinate system is thus

$$\int \int_A f\,dx\,dy = 2 \int \int_{A^+} f\,dx\,dy = 2 \int \int_K f|J|\,dr\ dp. \tag{A10}$$

We now turn our attention to the evaluation of the Jacobian J of the transformation

$$J = \begin{vmatrix} \dfrac{\partial X}{\partial r} & \dfrac{\partial Y}{\partial r} \\ \dfrac{\partial X}{\partial p} & \dfrac{\partial Y}{\partial p} \end{vmatrix} \tag{A11}$$

Substituting the appropriate partial derivatives into (A11) yields

$$J = \begin{vmatrix} \dfrac{-r}{2h} & \dfrac{r(4h^2 - r^2 + p^2)}{8h^2 Y} \\ \dfrac{p}{2h} & \dfrac{p(4h^2 + r^2 - p^2)}{8h^2 Y} \end{vmatrix}, \tag{A12}$$

where Y has been left in the partial derivatives for brevity. Solving for J and simplifying we obtain

$$|J| = \frac{2pr}{4hY} = \frac{2pr}{[8h^2(r^2+p^2) - (r^2-p^2)^2 - 16h^4]^{1/2}}. \tag{A13}$$

Clearly J is nonvanishing and well-defined except where $Y = 0$. As was shown in the derivation of the region of integration K, this corresponds to the boundary of K. Thus the transformation is well-defined inside of K and Eq. (A1) transforms to

$$R(t) = 2I_1 \int \int_K \sum_{i \in m(p)} \sum_{j \in n(r)} C_{A_{ij}(t)}(p, r)\ \frac{B(\theta_i, \phi_i)\ \overline{B}\ (\theta_j, \phi_j)}{L_i(p)\ L_j(r)}$$
$$\times\ \sigma(\theta_i', \phi_i, \theta_j', \phi_j)\ \frac{2pr\ dr\ dp}{[8h^2(p^2+r^2) - (p^2-r^2)^2 - 16h^4]^{1/2}}. \tag{A14}$$

END

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